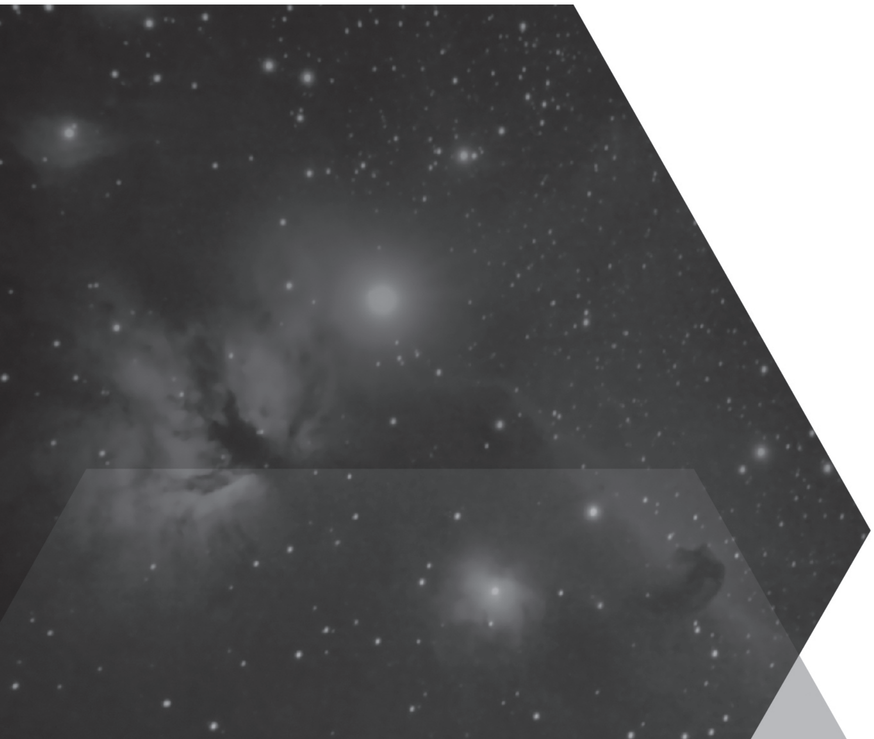
Edexcel GCSE

Maths

Higher Student Book
Answers



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Answers

Chapter 1 – Number: Basic number

Exercise 1A

1 **a** 6000
b 5 cans cost £1.95, so 6 cans cost £1.95.
32 = 5 × 6 + 2. Cost is £10.53.

2 **a** 288
b 16

3 **a** 38
b Coach price for adults = £8, coach price for juniors = £4, money for coaches raised by tickets = £12 400, cost of coaches = £12 160, profit = £240

4 (18.81...) Kirsty can buy 18 models.

5 £8.40 per year, 70p per copy

6 £450

7 15

8 3 weeks

9 £248.75

10 Gavin pays 2926.25 – 1840 = £1086.25

11 **a** Col is correct
b Abi has multiplied 30 × 50 as 150 instead of 1500. Baz has lined up the columns wrongly when adding. Instead of lining up the units he has lined up the first digits. Des has forgotten to add a zero on the second line of the multiplication, it should be 1530.

Exercise 1B

1 **a** 4.6 **b** 0.08 **c** 45.716
d 94.85 **e** 602.1 **f** 671.76
g 7.1 **h** 6.904 **i** 13.78
j 0.1 **k** 4.002 **l** 60.0

2 **a** 0.028 **b** 0.09 **c** 50.96 **d** 46.512

3 **a** 35, 35.04, 0.04 **b** 16, 18.24, 2.24
c 60, 59.67, 0.33 **d** 140, 140.58, 0.58

4 **a** 18 **b** 140 **c** 1.4 **d** 12 **e** 6.9

5 **a** 280 **b** 12 **c** 240 **d** 450 **e** 0.62

6 **a** 572
b **i** 5.72 **ii** 1.43 **iii** 22.88

7 **a** Incorrect as should end in the digit 2
b Incorrect since 9 × 5 = 45, so answer must be less than 45

8 300

9 **a** 27
b **i** 27 **ii** 0.027 **iii** 0.27

10 Mark bought a DVD, some jeans and a pen.

11 Headline A does not give the exact figure so does not convey any useful information. Headline B is accurate and records should be given accurately. Headline C may be correct but without the previous record does not convey any useful information.

Exercise 1C

1 **a** 50 000 **b** 90 000 **c** 30 000
d 200 **e** 0.5 **f** 0.006
g 0.3 **h** 10 **i** 0.05
j 1000

2 **a** 56 000 **b** 80 000 **c** 31 000
d 1.7 **e** 0.066 **f** 0.46
g 4.1 **h** 8.0 **i** 1.0
j 0.80

3 **a** 60 000 **b** 5300 **c** 89.7
d 110 **e** 9 **f** 1.1
g 0.3 **h** 0.7

4 **a** 65, 74 **b** 95, 149 **c** 950, 1499

5 Elsecar 750, 849; Hoyland 1150, 1249; Barnsley 164 500, 165 499

6 18 to 23 inclusive

7 1, because there could be 450 then 449

8 Donte has rounded to 2 significant figures or nearest 10 000

9 **a** Advantage – quick. Disadvantage – assumes 3 penguins a square metre which may not be accurate
b Advantage. Quite accurate as 5 by 5 is a big enough area to give a reliable estimate. Disadvantage – takes a long time.

Exercise 1D

1 **a** 60 000 **b** 120 000 **c** 10 000
d 15 **e** 140 **f** 100
g 200 **h** 0.08 **i** 0.09
j 45

2 **a** 5 **b** 25 **c** 3000
d 600 **e** 2000 **f** 5000
g 400 **h** 8000 **i** 4 000 000

3 30 × 90 000 = 2 700 000
600 × 8000 = 4 800 000
5000 × 4000 = 20 000 000
200 000 × 700 = 140 000 000

4 **a** 54 400 **b** 16 000

5 1400 million

6 His answer is correct but he had one too many zeros on each value, which cancel each other out. Matt wrote 600,000 rather than 60,000 and 2000 rather than 200. The two mistakes cancelled themselves out due to the zeros involved.

7 **a** Value of the money is about 66 000 000 × 0.2 = £13 200 000, so it is enough to buy the yacht.
b Weight is 66 000 000 × 5 = 330 000 000 grams = 330 tonnes, so they do not weigh as much as the yacht.

8 1420 000 000 000 ÷ 64 000 000 ≈ 22 200, so the National Debt per person is approximately £22 200.

Exercise 1E

1 **a** 35 000 **b** 15 000 **c** 960
d 5 **e** 1200 **f** 500

2 **a** 39 700 **b** 17 000 **c** 933
d 4.44 **e** 1130 **f** 550

3 **a** 1.74 m **b** 6 minutes **c** 240 g
d 83°C **e** 35 000 people **f** 15.5 miles
g 14 m^2

4 **a** 10 **b** 1 **c** 3

5 **a** 8.79 **b** 1.03 **c** 3.07

6 82°F, 5 km, 110 min, 43 000 people, 6.2 seconds, 67th, 1788, 15 practice walks, 5 seconds
The answers will depend on the approximations made. Your answers should be to the same order as these.

7 **a** £15 000 **b** £18 000 **c** £17 500

8 $1000

9 **a** 40 miles per hour **b** 10 gallons **c** £70

10 **a** 80 000 **b** 2000 **c** 1000 **d** 30 000
e 5000 **f** 2500 **g** 75 **h** 100

11 **a** 86 900 **b** 1760 **c** 1030 **d** 29 100
e 3960 **f** 2440 **g** 84.8 **h** 163

12 Approximately 500

13 £1 million pounds is 20 million 5p coins. 20 000 000 × 4.2 = 84 000 000 grams = 84 tonnes, so 5 lorries needed.

14 22.5° C – 18.2° C = 4.3 Celsius degrees

15 **a** **i** 27.571 428 57 **ii** 27.6
b **i** 16.896 516 39 **ii** 16.9
c **i** 18 672.586 16 **ii** 18 700

16 **a** 37.5 × 48.6 ≈ 40 × 50 = 2000 21.7 ×103.6 ≈ 20 × 100 = 2000 985 ÷ 0.54 ≈ 1000 ÷ 0.5 = 2000
b as both values are rounded down the actual answer must be bigger than 2000. The other two must be less than 2000.
c Pete is correct it is not possible to tell. 37.5 × 48.6 = 1822.5 985 ÷ 0.54 = 1824.074

17 149 000 000 ÷ 300 000 = 496.67 ≈ 500 seconds

18 **a** 58.9 × 4.8 ≈ 60 × 5 = 300
b Lower as both values are rounded up to get the estimate.

19 Macau's population density is approximately 710 000 times the population density of Greenland.

20 26.8 ÷ 3.1 ≈ 27 ÷ 3 = 9 36.2 ÷ 3.9 ≈ 36 ÷ 4 = 9. Second calculation must be biggest as first is smaller than 27 ÷ 3 and second is bigger than 36 ÷ 4.

Exercise 1F

1 **a** 12 **b** 9 **c** 6 **d** 13 **e** 15 **f** 14
g 16 **h** 10 **i** 18 **j** 17 **k** 8 or 16 **l** 21

2 4 packs of sausages and 5 packs of buns (or multiples of these)

3 30 seconds

4 12 minutes; Debbie will have run 4 laps; Fred will have run 3 laps.

5 1 + 3 + 5 + 7 + 9 = 25, 1 + 3 + 5 + 7 + 9 + 11 = 36, 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49, 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64

6 **a** −2 **b** −7 **c** −12 **d** −1 **e** −30

7 **a** 1 **b** 3 **c** 4 **d** 2 **e** −4

8 **a** 400 **b** 900 **c** 2500 **d** 0.25 **e** 16

9 **a** Student's own explanation
b 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105
c Adding consecutive pairs gives you square numbers.

10

	Square number	Factor of 56
Cube number	64	8
Multiple of 7	49	28

11 2, 3 and 12

12 **a** 1, 64, 729, 4096, 15 625
b 1, 8, 27, 64, 125
c $\sqrt{a^3} = a \times \sqrt{a}$
d Square numbers

13 **a** 0.2 **b** 0.5 **c** 0.6 **d** 0.9
e 1.5 **f** 2.1 **g** 0.8 **h** 0.7

14 The answers will depend on the approximations made. Your answers should be to the same order as these.
a 60 **b** 1500 **c** 150

Exercise 1G

1 **a** 84 = 2 × 2 × 3 × 7
b 100 = 2 × 2 × 5 × 5
c 180 = 2 × 2 × 3 × 3 × 5
d 220 = 2 × 2 × 5 × 11
e 280 = 2 × 2 × 2 × 5 × 7
f 128 = 2 × 2 × 2 × 2 × 2 × 2 × 2
g 50 = 2 × 5 × 5

2 **a** $84 = 2^2 \times 3 \times 7$ **b** $100 = 2^2 \times 5^2$
c $180 = 2^2 \times 3^2 \times 5$ **d** $220 = 2^2 \times 5 \times 11$
e $280 = 2^3 \times 5 \times 7$ **f** $128 = 2^7$
g $50 = 2 \times 5^2$

3 1, 2, 3, 2^2, 5, 2 × 3, 7, 2^3, 3^2, 2 × 5, 11, 2^2 × 3, 13, 2 × 7, 3 × 5, 2^4, 17, 2 × 3^2, 19, 2^2 × 5, 3 × 7, 2 × 11, 2^3, 23 × 3, 5^2, 2 × 13, 3^3, 2^2 × 7, 29, 2 × 3 × 5, 31, 2^5, 3 × 11, 2 × 17, 5 × 7, 2^2 × 3^2, 37, 2 × 19, 3 × 13, 2^3 × 5, 41, 2 × 3 × 7, 43, 2^2 × 11, 3^2 × 5, 2 × 23, 47, 2^4 × 3, 7^2, 2 × 5^2

4 **a** 2 is always the only prime factor
b 64, 128 **c** 81, 243, 729
d 256, 1024, 4096
e 3, 3^2, 3^3, 3^4, 3^5, 3^6; 4, 4^2, 4^3, 4^4, 4^5, 4^6

5 a $2 \times 2 \times 3 \times 5$ **b** $2^2 \times 3 \times 5$
c $120 = 2^3 \times 3 \times 5$, $240 = 2^4 \times 3 \times 5$, $480 = 2^5 \times 3 \times 5$

6 a $7^2 \times 11^2 \times 13^2$ **b** $7^3 \times 11^3 \times 13^3$
c $7^{10} \times 11^{10} \times 13^{10}$

7 Because 3 is not a factor of 40 so it does not divide exactly.

8 $a = 2$, $b = 7$

9 a $2ab$ a^2 $4b$ **b** $8a^3b$ $4a^3b^2$

Exercise 1H

1 a 20 **b** 56 **c** 6 **d** 28
e 10 **f** 15 **g** 24 **h** 30

2 They are the two numbers multiplied together.

3 a 8 **b** 18 **c** 12 **d** 30

4 No. The numbers have a common factor. Multiplying them together would mean using this factor twice, thus increasing the size of the common multiple. It would not be the least common multiple.

5 a 168 **b** 105 **c** 84 **d** 84
e 96 **f** 54 **g** 75 **h** 144

6 3 packs of cheese slices and 4 packs of bread rolls

7 a 8 **b** 7 **c** 4 **d** 16 **e** 14 **f** 9

8 a **ii** and **iii** **b** **iii**

9 18 and 24

10 a $6x^2y^2$ **b** xy

Exercise 1I

1 a 7 **b** –8 **c** –5 **d** – 11
e 11 **f** 6 **g** 8 **h** 8
I –2 **j** –1 **k** –9 **l** –5
m 5 **n** –9 **o** 8 **p** 0

2 a –15 **b** –14 **c** –24 **d** 6
e 14 **f** 2 **g** –2 **h** –8
i –4 **j** 3 **k** –24 **l** –10
m –18 **n** 16 **o** 36

3 a −9 **b** 16 **c** −3 **d** −32
e 18 **f** 18 **g** 6 **h** −4
i 20 **j** 16 **k** 8 **l** −48
m 13 **n** −13 **o** −8

4 a −2 **b** 30 **c** 15 **d** −27 **e** −7

5 a –9 **b** 3 **c** 1

6 a 16 **b** −2 **c** −12

7 −1 × 12, 1 × −12, −2 × 6, 2 × −6, −3 × 4, 3 × −4,

8 Any appropriate divisions

9 a −24 **b** 24 degrees **c** 3 × −6

10 13 × −6, −15 × 4, −72 ÷ 4, −56 ÷ −8

11 a 32°F and 212°F **b** –40°C = –40°F

12 –460°F

Exercise 1J

1 a –4 **b** −6 **c** 4 **d** 45 **e** 6 **f** 6

2 a 38 **b** 24 **c** −3 **d** –6 **e** −1 **f** 2
g −25 **h** 25 **i** 0 **j** −20 **k** 4 **l** 0

3 a (3 × −4) + 1 = −11 **b** −6 ÷ (−2 + 1) = 6
c (−6 ÷ −2) + 1 = 4 **d** 4 + (−4 ÷ 4) = 3
e (4 + −4) ÷ 4 = 0 **f** (16 − −4) ÷ 2 = 10

4 a 49 **b** −1 **c** −5 **d** −12

5 a 40 **b** 1 **c** 78 **d** 4

6 Possible answer: 3 × −4 ÷ 2

7 Possible answer: (2 − 4) × (7 − 3)

8 $(-4)^2 = -4 \times -4 = +16$, $-(4)^2 = -(4 \times 4) = -16$

9 (5 + 6) − (7 ÷ 8) × 9

10 –6

Review questions

1 10 weeks

2 16

3 270

4 a $3^2 \times 5 \times 7$ **b** 63

5 a 11.412 712 21 **b** 11.4

6 a 412.603252 **b** 400.5

7 a **iii** Prime numbers less than 20
b **i** 252 **ii** 3780 **iii** 18

8 a 10.663 418 78 **b** 11

9 1200

10 5

11 a 3.141 592 92 **b** 0.000 009%

12 a 7:30 pm (7:45pm on Town Hall clock)
b 6:00 pm on Tuesday (7:00pm on Town Hall clock)

13 a 15 120 **b** 12

14 a 90 **b** 240 **c** 6

15 27 and 36

16 a 2000
b Higher as top values rounded down and denominator rounded up.

17 a p and q are 2 and 5. r is 3 **b** 15

18 m = 5, n = 3

Chapter 2 – Number: Fractions, ratio and proportion

Exercise 2A

1 a $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{5}{24}$

e $\frac{2}{5}$ **f** $\frac{1}{6}$ **g** $\frac{2}{7}$ **h** $\frac{1}{3}$

2 $\frac{12}{30} = \frac{2}{5}$

3 $\frac{1}{5}$

4 $\frac{1}{2}$

5 Jon saves $\frac{30}{90} = \frac{1}{3}$

Matt saves, $\frac{35}{100} = \frac{7}{20}$ which is greater than $\frac{1}{3}$, so Matt saves the greater proportion of his earnings.

6 $\frac{13}{20} = \frac{65}{100}$ and $\frac{16}{25} = \frac{64}{100}$ so 13 out of 20 is the better mark.

7 $\frac{3}{8}$

8 $\frac{11}{24}$

9 $\frac{3}{7}$

10 $\frac{9}{22}$

Exercise 2B

1 **a** $\frac{8}{15}$ **b** $\frac{7}{12}$ **c** $\frac{11}{12}$
d $\frac{1}{10}$ **e** $\frac{1}{8}$ **f** $\frac{1}{12}$

2 Three-quarters of 68

3 **a** $4\frac{47}{60}$ **b** $\frac{41}{72}$ **c** $1\frac{109}{120}$ **d** $1\frac{23}{30}$

4 **a** $\frac{1}{6}$ **b** 30

5 No, one eighth is left, which is 12.5 cl, so enough for one cup but not two cups.

6 He has added the numerators and added the denominators instead of using a common denominator. Correct answer is $3\frac{7}{12}$.

7 Possible answer: The denominators are 4 and 5. I first find a common denominator. The lowest common denominator is 20 because 4 and 5 are both factors of 20. So I am changing the fractions to twentieths. One-quarter is the same as five-twentieths (multiplying numerator and denominator by 5). Two-fifths is the same as eight-twentieths (multiplying numerator and denominator by 4). Five-twentieths plus eight-twentieths = thirteen-twentieths.

8 $\frac{11}{20}$ of 900 = 495, $\frac{2}{11}$ of 495 = 90 left-handed boys. 900 – 495 = 405 girls. $\frac{2}{9}$ of 405 = 90 left-handed girls. 180 left-handed students altogether so 180 out of 900 = $\frac{1}{5}$.

9 $\frac{1}{5} + \frac{3}{8} = \frac{23}{40}$, so $\frac{17}{40}$ of the counters are yellow. $\frac{17}{40}$ of 600 = 255

10 **a** because $\frac{27}{40} + \frac{2}{5} = 1\frac{3}{40}$ which is greater than 1.

b $\frac{2}{5}$ of 200 = 80. $\frac{5}{8}$ of 80 = 50 women at least 40. $\frac{27}{40}$ of 200 = 135 members at least 40. 135 – 50 = 85 men at least 40. $\frac{3}{5}$ of 200 = 120, so 120 – 85 = 35 men under 40.

11 **a** $\frac{1}{5}$ is $\frac{8}{40}$. $\frac{3}{4}$ is $\frac{30}{40}$. Half-way between 8 and 30 is 19, so the mid-point fraction is $\frac{19}{40}$.

b Yes as the mid-point of any two numbers a and b is $(a + b) \div 2$ and adding the same denominator is the same thing as dividing by 2.

Exercise 2C

1 **a** $\frac{1}{6}$ **b** $\frac{3}{8}$ **c** $\frac{7}{20}$ **d** $\frac{3}{5}$
e $\frac{5}{12}$ **f** $2\frac{11}{12}$ **g** $3\frac{9}{10}$ **h** $3\frac{1}{3}$

2 **a** $\frac{3}{4}$ **b** $1\frac{1}{15}$ **c** 5 **d** $\frac{4}{9}$ **e** $1\frac{3}{5}$

3 **a** $\frac{1}{4}$ **b** 5 **c** $\frac{8}{3}$ **d** $\frac{4}{5}$

4 **a** $-\frac{1}{5}$ **b** 2 **c** $-\frac{9}{7}$ **d** $\frac{5}{3}$

5 $\frac{3}{8}$

6 $\frac{1}{8}$

7 40

8 $\frac{2}{5}$ of $6\frac{1}{2}$

9 £10.40

10 **a** $\frac{9}{32}$ **b** $\frac{256}{625}$

11 After 1 day $\frac{7}{8}$ of the water is left. On day 2, $\frac{1}{8} \times \frac{7}{8} = \frac{7}{64}$ is lost so total lost is $\frac{1}{8} + \frac{7}{64} = \frac{8}{64} + \frac{7}{64} = \frac{15}{64}$, so $1 - \frac{15}{64}$ is left = $\frac{49}{64}$

12 $50 \times 1\frac{1}{2}$ = 75 kg. 120 – 75 = 45, $45 \div 2\frac{1}{2}$ = 18, so 18 of the $2\frac{1}{2}$ kg bags are packed.

13 **a** 77% is about $\frac{3}{4}$. 243 is about 240, so $\frac{3}{4}$ of 240 = 180.

b Lower, as both estimates are lower than the original values.

Exercise 2D

1 **a** $1\frac{11}{20}$ **b** $1\frac{1}{4}$ **c** $1\frac{63}{80}$
d $\frac{11}{30}$ **e** $\frac{61}{80}$ **f** $\frac{167}{240}$

2 **a** $12\frac{1}{4}$ miles **b** $3\frac{1}{4}$ miles

3 **a** $6\frac{11}{20}$ **b** $8\frac{8}{15}$ **c** $11\frac{63}{80}$
d $3\frac{11}{30}$ **e** $7\frac{61}{80}$ **f** $4\frac{277}{396}$

4 **a** $-\frac{77}{1591}$ **b** Answer is negative

5 $18\frac{11}{12}$ cm

6 $\frac{5}{12}$ (anticlockwise) or $\frac{7}{12}$ (clockwise)

7 **a** $\frac{3}{5}$ **b** $\frac{27}{128}$ **c** $5\frac{2}{5}$
d $5\frac{1}{7}$ **e** $3\frac{9}{32}$ **f** $\frac{11}{18}$

8 **a** $8\frac{11}{20}$ **b** $65\frac{91}{100}$ **c** $52\frac{59}{160}$
d $2\frac{17}{185}$ **e** $2\frac{22}{103}$ **f** $7\frac{881}{4512}$

9 $18\frac{5}{12}$ m²

10 3

11 **a** $6 \times (1\frac{3}{4})^2 = 18\frac{3}{8}$ cm²
b $34\frac{14}{25} \div 6 = \frac{144}{25}$, $\sqrt{\frac{144}{25}} = \frac{12}{5} = 2\frac{2}{5}$ cm

12 $22 \div (2 \times \frac{22}{7}) = \frac{7}{2}$, $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38\frac{1}{2}$ cm²

13 Volume cuboid = $22\frac{11}{24}$ cm³, $22\frac{11}{24} \div (\frac{22}{7} \times \frac{4}{3}) =$
$\frac{343}{64}$, $\sqrt[3]{\frac{343}{64}} = 1\frac{3}{4}$ cm

14 After 1 day $\frac{7}{8}$ is left, after two days $\frac{49}{64}$ and after three days $\frac{343}{512}$ is left

15 $120 \times 4\frac{1}{2} = 540$. $175 \times 1\frac{1}{2} = 262\frac{1}{2}$. $540 - 262\frac{1}{2} = 277\frac{1}{2}$. $277\frac{1}{2} \div 2\frac{1}{2} = 111$ bags.

Exercise 2E

1 **a** 1.1 **b** 1.03 **c** 1.2 **d** 1.07 **e** 1.12

2 **a** 0.92 **b** 0.85 **c** 0.75 **d** 0.91 **e** 0.88

3 **a** 391 kg **b** 824.1 cm **c** 253.5 g
d £143.50 **e** 736 m **f** £30.24

4 **a** 731 m **b** 83.52 g **c** 360 cm
d 117 min **e** 81.7 kg **f** £37.70

5 448

6 No, as the total is £101. She will save £20.20, which is less than the £25 it would cost to join the club.

7 7% pay rise is an increase of £1925 per year which is better than £150 × 12 = £1800

8 **a** £6.125 (£6.13)
b $x \times 0.025$
c $y \div 1.175 \times 1.2$

9 Offer A gives 360 grams for £1.40, i.e. 0.388 pence per gram.
Offer B gives 300 grams for £1.12, i.e 0.373 pence per gram, so Offer B is the better offer.
Or Offer A is 360 for 1.40 = 2.6 g/p, offer B is 300 for 1.12 = 2.7 g/p, so offer B is better.

10 **c** Both the same as 1.05 × 1.03 = 1.03 × 1.05

11 **a** Shop A as 1.04 × 1.04 = 1.0816, so an 8.16% increase.

12 £425.25

13 0.9 × 1.1 = 0.99 (99%)

14 Area of original circle = 200.96
Enlarged area = 200.96 x 1.6 = 321.536
Enlarged radius = $\sqrt{321.536 \div 3.14}$ = 10.1192885125
% increase = 2.11928/8 × 100 = 26.49%

15 **a** Let r = 10. Approx formula gives V = 4000, actual gives V = 4188.79, 188.79 ÷ 4188.79 = 0.045 which is 4.5%
b The value is lower as $\frac{4}{3}$ × π is greater than 4 as π is 3.14.

Exercise 2F

1 **a** 25% **b** 60.6% **c** 46.3% **d** 12.5%
e 41.7% **f** 60% **g** 20.8% **h** 10%
i 1.9% **j** 8.3% **k** 45.5% **l** 10.5%

2 32%

3 6.5%

4 33.7%

5 **a** 49.2% **b** 64.5% **c** 10.6%

6 4.9%

7 90.5%

8 Stacey had the greater percentage increase.
Stacey: (20 − 14) × 100 ÷ 14 = 42.9%
Calum: (17 − 12) × 100 ÷ 12 = 41.7%

9 Yes, as 38 out of 46 is over 80% (82.6%)

10 Let z = 100. y = 75, x = 0.6 × 75 = 45, so x is 45% of z

11 Let z be 100, x = 60. If x is 75% of y, y = 80, so y is 80% of z.

12 30% of 4800 = 1440. 1.2 × 4800 = 5760. 70% of 5760 = 4032. (4032 – 1440) ÷ 1440 = 1.8, so the increase in numbers owning a mobile phone is 180%.

13 31 ÷ 26 = 1.19 which is a 19% increase. 31% is 5% more of the total votes cast than 26%

Review questions

1 £572

2 **a** 36 seconds
b i 25.2 seconds **ii** Eve **iii** Eve

3 £120

4 £576

5 **a** £9 **b** £13.20

6 **a** 0.875 **b** $\frac{11}{35}$ **c** $5\frac{1}{5}$

7 £322

8 $\frac{19}{40}$

9 $4\frac{1}{12}$

10 5

11 **a** $\frac{221}{71}$, $\frac{22}{7}$, $\frac{312}{99}$, $\frac{54}{17}$ **b** $\frac{22}{7}$

12 28%

13 77%

14 25%

15 For bag A P(red) = 0.1875 and for bag B P(red) = 0.186 so Tomas is wrong.

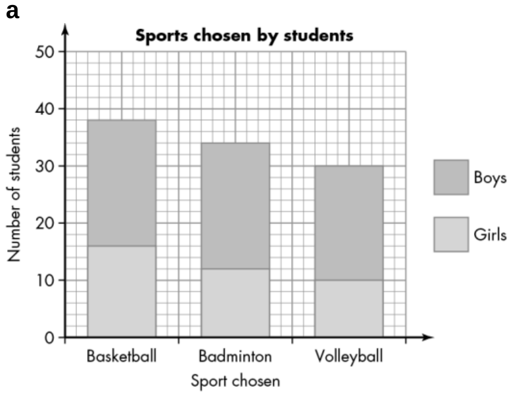
16 13%

17 **a** 150 men, 100 women **b** 12%

Chapter 3 – Statistics: Statistical diagrams and averages

Exercise 3A

1 **a**



b 16 **c** 42

2 Pie charts with these angles:
a 36°, 90°, 126°, 81°, 27°
b 90°, 108°, 60°, 78°, 24°
c 168°, 52°, 100°, 40°

3 **a** Pictogram with suitable key
b Bar chart correctly labelled
c Vertical line chart correctly labelled
d Pie chart with these angles: 60°, 165°, 45°, 15°, 75° and correctly labelled
e Vertical line chart. It shows the frequencies, the easiest one to draw and comparisons can be made; or other chart with a valid supporting reason given.

4 **a** 36
b Pie charts with these angles: 50°, 50°, 80°, 60°, 60°, 40°, 20°
c Student's bar chart.
d Bar chart, because easier to make comparisons; or other chart stated with a valid supporting reason

5 **a** Pie charts with these angles: 124°, 132°, 76°, 28°
b Split of total data seen at a glance.

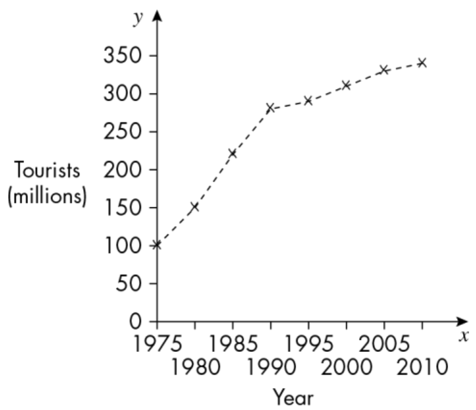
6 **a** 55° **b** 22 **c** $33\frac{1}{3}$%

7 **a** Pie charts with these angles:
Strings: 36°, 118°, 126°, 72°, 8°
Brass: 82°, 118°, 98°, 39°, 23°
b Overall, the strings candidates did better, as a smaller proportion obtained lower grades. A higher proportion of Brass candidates scored very good grades.

8 Work out the angle for 'Don't know' = 40°, so P(Don't know) = $\frac{40}{360}$° = $\frac{1}{9}$

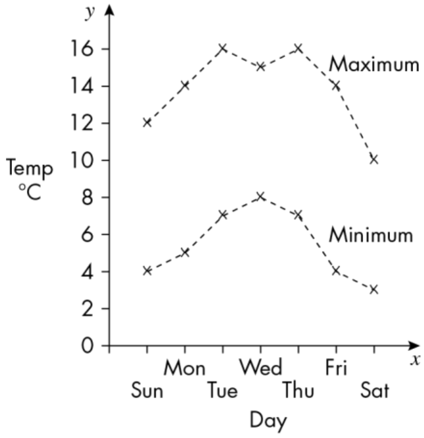
Exercise 3B

1 **a**



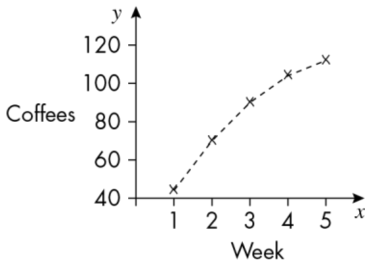
b About 328 million
c Between 1980 and 1985
d Rising steeply at first, but then leveling off. Rise in living standards, cheaper flights, more package holidays

2 **a**



b Smallest difference Wednesday and Saturday (7°), greatest difference Friday (10°)

3 **a**



b about 120

c The same people keep coming back and tell others, but new customers each week become more difficult to find.

4 No, you cannot extrapolate the data or the data is likely to change after 5 weeks

5 All the temperatures were presumably higher than 20 °C.

Exercise 3C

1 **a** 47 **b** 53 **c** 55 **d** 65

2 Mode

3 **a**

0	8	9	9	9		
1	2	2	3	7	7	8
2	0	1	2	5		

Key 0 | 8 represents 8 text messages

b 17

c 15

4 Three possible answers: 12, 14, 14, 16, 18, 20, 24; or 12, 14, 14, 16, 18, 22, 24; or 12, 14, 14, 16, 20, 22, 24

5 **a** Median (mean could be unduly influenced by results of very able and/or very poor candidates)

b Median (mean could be unduly influenced by pocket money of students with very rich or generous parents)

c Mode (numerical value of shoe sizes irrelevant, just want most common size)

d Median (mean could be distorted by one or two extremely short or tall performers)

e Mode (the only way to get an ‘average’ of non-numerical values)

f Median (mean could be unduly influenced by very low weights of premature babies)

6 **a** 20 **b** 25 **c** 46 **d** 43

e The boys did better as they had a higher median and their marks were less spread out.

7 **a i** £20 000 **ii** £28 000 **iii** £34 000

b A 6% rise would increase the mean salary to £36 040, a £1500 pay increase would produce a mean of £35 500.

8 **a** Median **b** Mode **c** Mean

9 Tom – mean, David – median, Mohammed – mode

10 11.6

11 42.7 kg

12 24

Exercise 3D

1 **a i** 7 **ii** 6 **iii** 6.4

b i 8 **ii** 8.5 **iii** 8.2

2 **a** 1280 **b** 1.9 **c** 0 **d** 328

3 **a** 2.2, 1.7, 1.3 **b** Better dental care

4 **a** 50 **b** 2 **c** 2.8

5 **a** Roger 5, Brian 4 **b** Roger 3, Brian 8

c Roger 5, Brian 4 **d** Roger 5.4, Brian 4.5

e Roger, smaller range **f** Brian, better mean

6 **a** 40 **b** 7 **c** 3 **d** 2 **e** 2.5

f the mode, 3 **g** 2.4

7 5

8 The total frequency could be an even number where the two middle numbers have an odd difference.

9 **a** 34

b $x + 80 + 3y + 104 = 266$, so $x + 3y = 82$

c $x = 10$, $y = 24$

d 2.5

Exercise 3E

1 **a i** $30 < x \leq 40$ **ii** 29.5

b i $0 < y \leq 100$ **ii** 158.3

c i $5 < z \leq 10$ **ii** 9.43

d i 7–9 **ii** 8.41

2 **a** $100 < m \le 120$ **b** 10.86 kg **c** 108.6 g

3 **a** $175 < h \le 200$ **b** 31% **c** 193.3 hours
d No the mean was under 200 and so was the mode.

4 24

5 **a** Yes, average distance is 11.7 miles per day.
b Because shorter runs will be run at a faster speed, which will affect the average.
c Yes, for example: 22miles – 1mile = 21 miles

6 Soundbuy; average increases are Soundbuy 17.7p, Springfields 18.7p, Setco 18.2p

7 **a** 160 **b** 52.6 minutes
c Modal group **d** 65%

8 The first 5 and the 10 are the wrong way round.

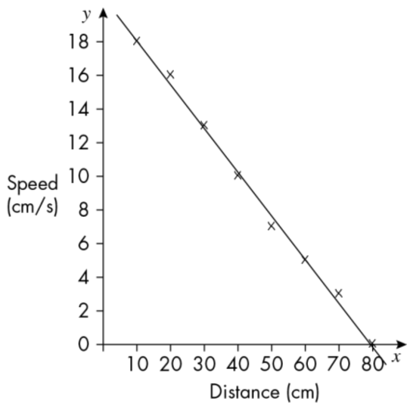
9 Find the midpoint of each group, multiply that by the frequency and add those products. Divide that total by the total frequency.

10 **a** Yes, as total in first two columns is 50, so median is between 39 and 40.
b He could be correct, as the biggest possible range is 69 – 20 = 49, and the lowest is 60 – 29 = 31.

Exercise 3F

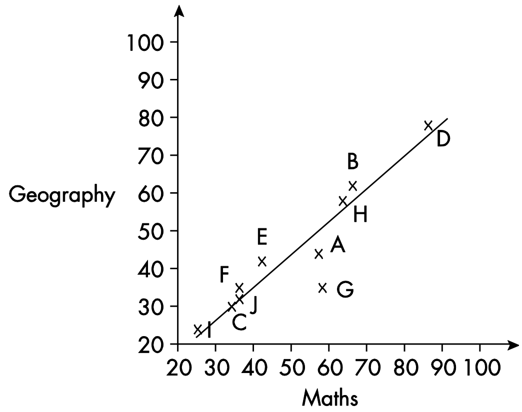
1 **a** Positive correlation, time taken increases with the number of press-ups
b strong negative correlation, you complete a crossword more quickly as you get older
c No correlation, speed of cars on M1 is not related to the temperature
d weak positive correlation, older people generally have more money saved in the bank

2 **a** and **b**



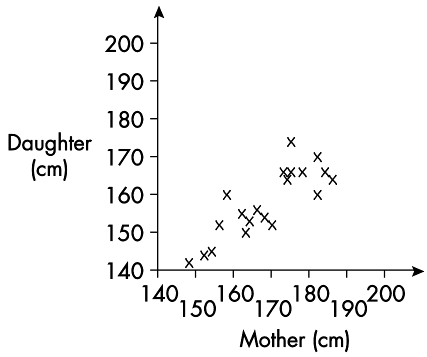
c about 19 cm/s
d about 34 cm

3 **a** and **b**



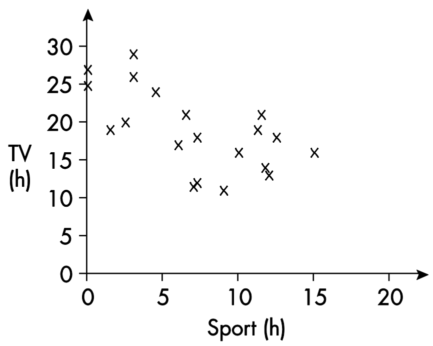
c Greta
d about 65
e about 75

4 **a**



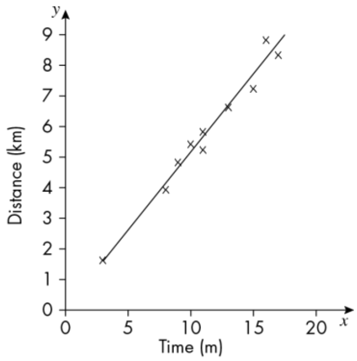
b Yes, as positive correlation is observed.

5 **a**



b Very weak negative correlation, so cannot draw a line of best fit or predict the value

6 **a** and **b**



c about 2.4 km
d about 8 minutes
e You cannot extrapolate values from a scatter diagram or the data may change for longer journeys

7 About 23 mph

8 Points showing a line of best fit sloping down from top left to bottom right

Review questions

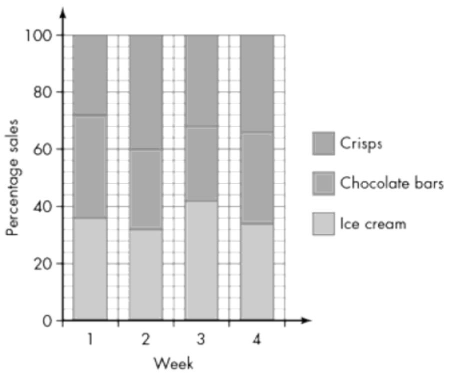
1 **a** Grade 7
b $\frac{100}{360}$ or $\frac{5}{18}$
c i 48 **iii** 216
d e.g. pie charts show proportions or they are percentages, not actual numbers or do not know how many students, etc.

2 43.7 matches

3 **a** $10 < t \leq 20$
b $10 < t \leq 20$
c 19 minutes

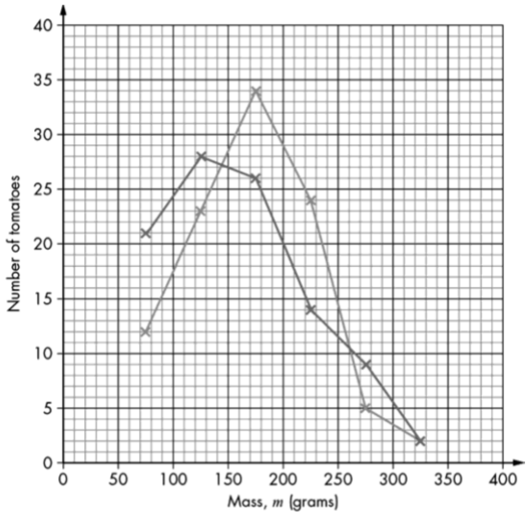
4 **a** because over half the students have more than £10 pocket money, so the mean must be more than £10
b £11.13

5



6 **a** $100 < m \leq 150$
b $150 < m \leq 200$
c 159 g

d



e

mass, *m* (grams)	**Margot's tomatoes**
$50 < m \leq 100$	12
$100 < m \leq 150$	23
$150 < m \leq 200$	34
$200 < m \leq 250$	24
$250 < m \leq 300$	5
$300 < m \leq 350$	2

f

mass, *m* (grams)	**Margot's tomatoes**	**mid point *x***	***x* × *m***
$50 < m \leq 100$	12	75	900
$100 < m \leq 150$	23	125	2875
$150 < m \leq 200$	34	175	5950
$200 < m \leq 250$	24	225	5400
$250 < m \leq 300$	5	275	1375
$300 < m \leq 350$	2	325	650
totals	100		17150

estimate for the mean = 171.5 g

g on average Tom's tomatoes had a smaller mass and were therefore probably smaller in size

7 **a i** Diagram C **ii** Diagram A **iii** Diagram B
b Diagram A: strong negative correlation, diagram B: no correlation, diagram C: strong positive correlation

8 **a/b** Student's graph as follows: Time on horizontal axis from 0 to 20 and Distance (km) on vertical axis from 0 to 10 with the following points plotted: (3, 1.7) (17, 8.3) (11, 5.1) (13, 6.7) (9, 4.7) (15, 7.3) (8, 3.8) (11, 5.7) (16, 8.7) (10, 5.3) and with line of best fit drawn.
c/d answers depend on student's line of best fit

Chapter 4 – Algebra: Number and sequences

Exercise 4A

1 **a** 11111 × 11111 = 123 454 321, 111111 × 111111 = 12 345 654 321
b 99999 × 99999 = 9 999 800 001, 999999 × 999999 = 999 998 000 001

2 **a** $7 \times 8 = 7^2 + 7$, $8 \times 9 = 8^2 + 8$
b 50 × 51 = 2550, 60 × 61 = 3660

3 **a** $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5^2$, $1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36 = 6^2$
b $21 + 23 + 25 + 27 + 29 = 125 = 5^3$, $31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$

4 **a** 1 + 6 + 15 + 20 + 15 + 6 + 1 = 64, 1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 128
b 12 345 679 × 45 = 555 555 555, 12 345 679 × 54 = 666 666 666

5 **a** $1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2 = 100$, $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 225$
b $36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$, $55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$

6 **a** 12 345 678 987 654 321
b 999 999 998 000 000 001
c $12^2 + 12$
d 8190
e $81 = 9^2$
f $512 = 8^3$
g 512
h 999 999 999
i $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)^2 = 2025$

7 1 + 500 = 501, 2 + 499 = 501, …. 250 + 251 = 501, 250 × 501 = 125250

Exercise 4B

1 **a** 21, 34: add previous 2 terms
b 49, 64: next square number
c 47, 76: add previous 2 terms

2 15, 21, 28, 36

3 61, 91, 127

4 $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}$

5 **a** 6, 10, 15, 21, 28
b It is the sums of the natural numbers, or the numbers in Pascal's triangle or the triangular numbers.

6 **a** 2, 6, 24, 720 **b** 69!

7 364: Daily totals are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78 (these are the triangular numbers). Cumulative totals are: 1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364.

8 X. There are 351 (1 + 2 + ... + 25 + 26) letters from A to Z. 3 × 351 = 1053. 1053 − 26 = 1027, 1027 − 25 = 1002, so, as Z and Y are eliminated, the 1000th letter must be X.

9 29 and 41

10 No, because in the first sequence, the terms are always one less than in the 2nd sequence

11 $4n - 2 = 3n + 7$ rearranges as $4n - 3n = 7 + 2$, so $n = 9$

Exercise 4C

1 **a** 13, 15, $2n + 1$ **b** 25, 29, $4n + 1$
c 33, 38, $5n + 3$ **d** 32, 38, $6n - 4$
e 20, 23, $3n + 2$ **f** 37, 44, $7n - 5$
g 21, 25, $4n - 3$ **h** 23, 27, $4n - 1$
i 17, 20, $3n - 1$ **j** –8, –18, $42 - 10n$
k 4, 0, $24 - 4n$ **l** –1, –6, $29 - 5n$

2 **a** $3n + 1$, 151 **b** $2n + 5$, 105 **c** $5n - 2$, 248
d $4n - 3$, 197 **e** $8n - 6$, 394 **f** $n + 4$, 54
g $5n + 1$, 251 **h** $8n - 5$, 395 **i** $3n - 2$, 148
j $3n + 18$, 168 **k** $47 - 7n$, –303 **l** $41 - 8n$, – 359

3 **a** 33rd **b** 30th **c** 100th = 499

4 **a** **i** $4n + 1$ **ii** 401 **iii** 101, 25th
b **i** $2n + 1$ **ii** 201
iii 99 or 101, 49th and 50th
c **i** $3n + 1$ **ii** 301 **iii** 100, 33rd
d **i** $2n + 6$ **ii** 206 **iii** 100, 47th
e **i** $4n + 5$ **ii** 405 **iii** 101, 24th
f **i** $5n + 1$ **ii** 501 **iii** 101, 20th
g **i** $3n - 3$ **ii** 297 **iii** 99, 34th
h **i** $6n - 4$ **ii** 596 **iii** 98, 17th
i **i** $205 - 8n$ **ii** –595 **iii** 101, 13th
j **i** $227 - 2n$ **ii** 27 **iii** 99 or 101, 64th and 63rd

5 **a** $\frac{2n+1}{3n+1}$
b Getting closer to $\frac{2}{3}$ $(0.\dot{6})$
c **i** 0.667 774 (6dp) **ii** 0.666 778 (6dp)
d 0.666 678 (6dp), 0.666 667 (6dp)

6 **a** $\frac{4n-1}{5n+1}$
b Getting closer to $\frac{4}{5}$ (0.8)
c **i** 0.796 407 (6dp) **ii** 0.799 640 (6dp)
d 0.799 964 (6dp), 0.799 9996 (7dp)

7 **a** £305 **b** £600 **c** 3 **d** 5

8 **a** $\frac{3}{4}, \frac{5}{7}, \frac{7}{10}$
b **i** 0.666 666 777 8 **ii** $\frac{2}{3}$
c For n, $\frac{2n+1}{3n+1} \approx \frac{2n}{3n} = \frac{2}{3}$

9 **a** $8n + 2$ **b** $8n + 1$ **c** $8n$ **d** £8

10 **a** Sequence goes up in 2s; first term is 2 + 29
b $n + 108$
c Because it ends up as $2n \div n$
d 79th

11 If there was a common term then for some value of n the expressions would be equal i.e. $2n = 2n - 1$, Subtracting $2n$ from both sides gives $0 = -1$, which is impossible.

12 Difference is 19 – 10 = 9. 9 ÷ 3 = 3 so $A = 3$. $3 \times 5 + b = 10$, $b = -5$

Exercise 4D

1 **a** Even,

+	Odd	Even
Odd	Even	Odd
Even	Odd	Even

b Odd,

×	Odd	Even
Odd	Odd	Even
Even	Even	Even

2 **a** $1 + 3 + 5 + 7 = 16 = 4^2$, $1 + 3 + 5 + 7 + 9 = 25 = 5^2$
b **i** 100 **ii** 56

3 **a** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
b because odd + odd = even, odd plus even = odd and even + odd = odd.
c **i** a + 2b, 2a + 3b, 3a + 5b, 5a + 8b, 8a + 13b
ii coefficient of a odd and b even, a even and b odd, both odd, etc.

4 **a** Even **b** Odd **c** Odd
d Odd **e** Odd **f** Odd
g Even **h** Odd **i** Odd

5 **a** Odd or even **b** Odd or even
c Odd or even **d** Odd
e Odd or even **f** Even

6 **a** **i** Odd **ii** Even **iii** Even
b Any valid answer, e.g. $x(y + z)$

7 **a** 64, 128, 256, 512, 1024
b **i** $2^n - 1$ **ii** $2^n + 1$ **iii** 3×2^n

8 **a** The number of zeros equals the power.
b 6
c **i** $10^n - 1$ **ii** 2×10^n

9 **a** 125, 216
b 1 + 8 = 9, 1 + 8 + 27 = 36, 1 + 8 + 27 + 64 = 100… the answers are square numbers

10 **a** 28, 36, 45, 55, 66
b **i** 210 **ii** 5050
c You get the square numbers.

11 **a** **i** If n is odd, $n + 1$ is even.
If n is even, $n + 1$ is odd.
Even times odd is always even.
ii $2n$ must be even, since it has a factor of 2, so $2n + 1$ must be odd.
b Odd
Odd
Even
Even
Odd
c $(2n + 1)^2 = 4n^2 + 4n + 1$
or $(2n)^2 = 4n^2$
$4n^2 + 4n$ is even so adding 1 makes it odd
$4n^2$ is $2 \times 2n^2$ which is even

12 11th triangular number is 66, 18th triangular number is 171

13 **a** 36, 49, 64, 81, 100
b **i** $n^2 + 1$ **ii** $2n^2$ **iii** $n^2 - 1$

14 **a** 6, 24, 96, 384, 1536
b 21, 147, 1029, 7203, 50 421
c 2, 10, 50, 250, 1250
d 6, 60, 600, 6000, 60 000
e 54, 162, 486, 1458, 4374

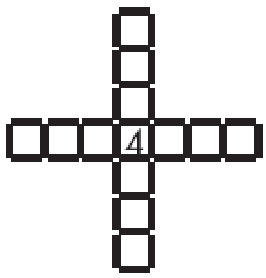
15 **a** $3 \times 2^{n-1}$ **b** $5 \times 4^{n-1}$
c $20 \times 5^{n-1}$ or 4×5^n
d $21 \times 3^{n-1}$ or 7×3^n
e $24 \times 8^{n-1}$ or 3×8^n

16 2 as all other primes are odd, so the sum of two of them will be even, so could not be a prime.

17 **a** There are many answers, 5 + 31 = 36, 7 + 29 = 36, 2 + 47 = 49 etc.
b There are many answers, 49 – 36 = 13, 81 – 64 = 17

Exercise 4E

1 **a**



b $4n - 3$
c 97
d 50th diagram

2 **a**

b $2n + 1$
c 121 **d** 49th set

3 **a** 18 **b** $4n + 2$ **c** 12

4 **a** **i** 24 **ii** $5n - 1$ **iii** 224
b 25

5 **a** 5, 8, 11, 14
b **i** 20 cm **ii** $(3n + 2)$ cm **iii** 152 cm
c 332

6 **a** **i** 20 **ii** 162
b 79.8 km

7 **a** **i** 14 **ii** $3n + 2$ **iii** 41
b 66

8 **a** **i** 5 **ii** n **iii** 18
b 20 tins

9 a 2^n

b i $100 \times 2^{n-1}$ ml **ii** 1600 ml

c Next sizes after super giant are 3.2*l*, 6.4*l* and 12.8*l* with weights of 3.2 kg, 6.4 kg and 12.8 kg, so the largest size is 6.4 litres.

10 The *n*th term is $\left(\frac{3}{4}\right)^n$, so as *n* gets very large, the unshaded area gets smaller and smaller and eventually it will be zero; so the shaded area will eventually cover the triangle.

11 Yes, as the number of matches is 12, 21, 30, 39, … which is $9n + 3$; so he will need $9 \times 20 + 3 = 183$ matches for the 20th step and he has $5 \times 42 = 210$ matches.

12 a 20 **b** 120

13 Alex's answer gives $4(n + 2) = 4n + 8$
Colin's method gives $4n + 4$
Ed's method gives $4(n + 1) = 4n + 4$
Gail's method gives $2 \times n + 2(n + 2) = 2n + 2n + 4 = 4n + 4$
Linear sequence is 8 12 16 20 …. Which has an *n*th term of $4n + 4$ so they are all valid methods except for Alex who forgot that the corners overlap and should have taken the 4 overlapping corners away to get $4n + 8 - 4 = 4n + 4$

Exercise 4F

1 a i 34, 43 **ii** goes up 3, 4, 5, 6, etc.
b i 24, 31 **ii** goes up 1, 2, 3, 4, etc.
c i 54, 65 **ii** goes up 5, 6, 7, 8, etc.
d i 57, 53 **ii** goes down 10, 9, 8, 7, etc.

2 a 4, 7, 12, 19, 28 **b** 2, 8, 18, 32, 50
c 2, 6, 12, 20, 30 **d** 4, 9, 16, 25, 36
e 2, 8, 16, 26, 38 **f** 4, 7, 14, 25, 40

3 a $2n + 1$ **b** n
c $n(2n + 1) = 2n^2 + n$
d $2n^2 + n + 1$

4 a n **b** $n + 1$
c $n(n + 1)$ **d** 9900 square units

5 a Yes, constant difference is 1 **b** No
c Yes, constant difference is 2 **d** No
e Yes, constant difference is 1 **f** No

6 a $4n + 4$ **b** n^2
c $n^2 + 4n + 4$ **d** $n^2 + 4n + 4$
e The sides of the large squares are of length $n + 2$ so the total number of squares is $(n + 2)^2$ which is the same answer as **c**.

7 a Table 10, 15, 21; 6, 10, 15; 16, 25, 36
b i 45 **ii** 100

8 $n^2 + 2n - 3 = n^2 + n + 3$, gives $n = 6$. Substituting gives 45 for both expressions.

9 a Sequences are 4, 7, 14, 25, 40, 59, 82, … and 4, 11, 20, 31, 44, 59, 76, … so 59 is the next common term.

b 59 is the 6th term in each sequence so substitute 6 into each expression. This will give 59 in both cases.

10 a There are many answers, for example $a = -3$ and $b = 1$.
b The only solution is $c = 2$ and $d = -3$

11 All values of *n* from 1 to 39 give a prime number. $n = 40$ gives 1681 which equals 41×41

12 a $(n + 1)(n - 1) = n^2 + n - n - 1 = n^2 - 1$
b $n^2 - 1$ as $50 \times 50 - 1$ is easy to work out but 51×49 isn't
c $(n + 1)(n - 1)$ as 100×98 is easy to work out but $99^2 - 1$ isn't.

Exercise 4G

1 a i 36, 49 **ii** n^2
b i 35, 48 **ii** $n^2 - 1$
c i 38, 51 **ii** $n^2 + 2$
d i 39, 52 **ii** $n^2 + 3$
e i 34, 47 **ii** $n^2 - 2$
f i 35, 46 **ii** $n^2 + 10$

2 a i 37, 50 **ii** $(n + 1)^2 + 1$
b i 35, 48 **ii** $(n + 1)^2 - 1$
c i 41, 54 **ii** $(n + 1)^2 + 5$
d i 50, 65 **ii** $(n + 2)^2 + 1$
e i 48, 63 **ii** $(n + 2)^2 - 1$

3 a i $n^2 + 4$ **ii** 2504
b i $3n + 2$ **ii** 152
c i $(n + 1)^2 - 1$ **ii** 2600
d i $n(n + 4)$ **ii** 2700
e i $n^2 + 2$ **ii** 2502
f i $5n - 4$ **ii** 246

4 a $2n^2 - 3n + 2$ **b** $3n^2 + 2n - 3$
c $\frac{1}{2}n^2 + \frac{5}{2}n + 1$ **d** $\frac{1}{2}n^2 + 4\frac{1}{2}n - 2$
e $\frac{1}{2}n^2 + 1\frac{1}{2}n + 6$ **f** $\frac{1}{2}n^2 + 1\frac{1}{2}n + 2$

5 $6n^2$

6 a 26 **b** $1\frac{1}{2}n^2 + \frac{1}{2}n$ **c** 8475

7 a 45
b *n*th term is $\frac{1}{2}n^2 + \frac{1}{2}n$ so $\frac{1}{2} \times 15 \times 15 + \frac{1}{2} \times 15 = 120$, so no.

8 Front face is n^2, sides faces are $n \times (n + 1) = n^2 + n$ so total surface area is
$2 \times n^2 + 4 \times (n^2 + n) = 6n^2 + 4n$.

9 Sequence is 1, 7, 19, 37. *n*th term is $3n^2 - 3n + 1$ so the 100th hexagonal number is 29 701.

10 a Taking the height first. There are $n + 1$ strips m feet long. That is $m(n + 1)$ in total.
Taking the width. There are $m + 1$ strips n feet long. That is $n(m + 1)$ in total
$m(n + 1) + n(m + 1) = mn + m + mn + n = 2mn + m + n$
b Taking the nails across a width strip. There are $n + 1$ lots of 2 nails which is $2(n + 1)$.
There are $m + 1$ width strips, so the total is $2(n + 1)(m + 1)$.

Review questions

1 No. Sequence is 7, 10, 13, 16, 19, 22, 25, 28, … so the first 3 odd terms are prime but 25 is not prime.

2 **a** $4n + 1$
b Not odd
c 28th term is 113

3 nth term is $5n + 1$. $5 \times 150 + 1 = 751$

4 **a** $6n + 3$
b No, $3n + 2$ generates the sequence 5, 8, 11, 14, 17, 20, 23, … so the even terms of this sequence are always 1 less than the terms of the original sequence

5 **a** $2 \times 3^{n-1}$ **b** Not an even number

6 **a** $5 \times 6^{n-1}$ **b** 8th term is 1 399 680

7 **a** The first five terms in the sequence are –27, –21, –11, 3, 21. Of these terms, 3 is a prime number.
b When $n = 29$ the expression can be factorised as $29(2 \times 29 - 1)$ so is not a prime number

8 **a** 4, 9, 18, 31, 48 **b** 2, 2, 3, 5, 8

9 $n = 1$ $(n - 1) = 0$, $n = 2$ $(n - 2) = 0$, $n = 3$ $2(3 - 1)(3 - 2) \div 5 = 2 \times 2 \times 1 \div 5 = 0.8$

10 $2n^2 - 2n + 3$

11 **a** nth term is $n^2 + 2n$, $12 \times 12 + 2 \times 12 = 168$, so yes he has enough squares
b $40 \times 40 + 2 \times 40 = 1680$

12 $2n^2 - n$, $2 \times 20^2 - 20 = 780$

13 The sequence of dots is 5, 15, 30, 50,…

n	0	1	2	3	4
c	0	5	15	30	50
$a + b$	5	10	15	20	
$2a$	5	5	5		

$a = 2\frac{1}{2}$, $b = 2\frac{1}{2}$ and $c = 0$,

so the nth term is $2\frac{1}{2}n^2 + 2\frac{1}{2}n$

$2\frac{1}{2} \times 50^2 + 2\frac{1}{2} \times 50 = 6375$

Chapter 5 – Ratio and proportion and rates of change: Ratio and proportion

Exercise 5A

1 **a** 1 : 3 **b** 3: 4 **c** 2 : 3 **d** 2 : 3
e 2 : 5 **f** 2 : 5 **g** 5 : 8 **h** 25 : 6

2 **a** 1 : 3 **b** 3 : 2 **c** 5 : 12
d 8 : 1 **e** 17 : 15 **f** 25 : 7
g 4 : 1 **h** 5 : 6 **i** 1 : 24

3 $\frac{7}{10}$

4 $\frac{2}{5}$

5 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$

6 7 : 3

7 2 : 1

8 **a** Fruit crush $\frac{1}{6}$, lemonade $\frac{5}{6}$

Fruit Crush	1.25	1	0.2	0.4	0.5
Lemonade	6.25	5	1	2	2.5

b 0.4 litres **c** 2.5 litres

9 **a** $\frac{1}{2}$ **b** $\frac{7}{20}$ **c** $\frac{3}{20}$

10 Sugar $\frac{5}{22}$, flour $\frac{3}{11}$, margarine $\frac{2}{11}$, fruit $\frac{7}{22}$

11 4

12 1 : 4

13 **a** 5 : 3 : 2 **b** 20

14 $13\frac{1}{2}$ litres

15 1 : 1 : 1

Exercise 5B

1 **a** 160 g, 240 g **b** 80 kg, 200 kg
c 150, 350 **d** 950 m, 50 m
e 175 min, 125 min **f** £20, £30, £50
g £36, £60, £144 **h** 50 g, 250 g, 300 g
i £1.40, £2, £1.60 **j** 120 kg, 72 kg, 8 kg

2 **a** 175 **b** 30%

3 **a** 40% **b** 300 kg

4 21

5 **a** Mott: no, Wright: yes, Brennan: no, Smith: no, Kaye: yes
b For example: W26, H30; W31, H38; W33, H37

6 **a** 1 : 400 000 **b** 1: 125 000 **c** 1 : 250 000
d 1 : 25 000 **e** 1 : 20 000 **f** 1 : 40 000
g 1 : 62 500 **h** 1 : 10 000 **i** 1 : 60 000

7 **a** $\frac{1}{2}$ km or 500m
b 78 cm ≈ 39 km. 39 ÷ 15 ≈ 2.6. 2.6 hours = 2 h 36 m. Plus 30 mins is 3 h 06 m so he should be back at about 12.06 pm

8 **a** Map A 1 : 250 000, Map B 1 : 1 000 000
b 2 km
c 1.2 cm
d 4.8 cm

9 **a** 1 : 1.6 **b** 1 : 3.25 **c** 1 : 1.125
d 1 : 1.44 **e** 1 : 5.4 **f** 1 : 1.5
g 1 : 4.8 **h** 1 : 42 **i** 1 : 1.25

10 Diesel : Petrol = 60 : 90. $\frac{1}{5}$ of 60 = 12. $\frac{4}{9}$ of 90 = 40. Total red cars = 52 which is more than 150 ÷ 3 = 50 so Yes.

11 a 4 : 3 **b** 90 miles
c Both arrive at the same time.

12 0.4 metres

13 13 – 9 = 4. 4 ÷ 5 = 0.8. 2 × 0.8 = 1.6, 9 + 1.6 = 10.6

14 Athos has 3 more parts than Zena. 24 ÷ 3 = 8, so 1 part is 8. Zena has 8 marbles.

Exercise 5C

1 a 3 : 2 **b** 32 **c** 80

2 a 100 **b** 160

3 0.4 litres

4 Jamie has 1.75 pints, so he has enough.

5 8100

6 296

7 Kevin £2040, John £2720

8 b C **c** F **d** T **e** T

9 51

10 100

11 40 ml

12 a 160 cans **b** 48 cans

13 a Lemonade 20 litres, ginger 0.5 litres
b This one, in part a there are 50 parts in the ratio 40 : 9 : 1, so ginger is $\frac{1}{50}$ of total amount; in part b there are 13 parts in the ratio 10 : 2 : 1, so ginger is $\frac{1}{13}$ of total amount. $\frac{1}{13} > \frac{1}{50}$

14 a Will as his multiple of 10 is also a multiple of 9
b Zeke has rounded off to 1 dp and and Yoko has rounded off to 2 dp. They have not used a recurring decimal notation.

15 54

Exercise 5D

1 60 g

2 £5.22

3 45

4 £6.72

5 a £312.50 **b** 8

6 a 56 litres **b** 350 miles

7 a 300 kg **b** 9 weeks

8 40 seconds

9 a i 100 g, 200 g, 250 g, 150 g
ii 150 g, 300 g, 375 g, 225 g
iii 250 g, 500 g, 625 g, 375 g
b 24

10 I can buy four packs (24 sausages) from Peter (£9.20)
I can only buy two packs (20 sausages) from Paul (£7)
I should use Peter's shop to get the most sausages for £10.

11 400 ÷ 10 = 40 loaves needed. 1.8 kg ÷ 3 = 0.6 kg per loaf, so 40 × 0.6 = 24 kg of flour.

12 4 buns and 5 cakes

13 11 minutes 40 seconds + 12 minutes = 23 minutes 40 seconds

14 Possible answer:
30 g plain flour (rounding to nearest 10 g)
60 ml whole milk (rounding to nearest 10 ml)
1 egg (need an egg)
1 g salt (nearest whole number)
10 ml beef dripping or lard (rounding to nearest 10 ml)

15 30 litres

Exercise 5E

1 a £4.50 for a 10-pack
b £1.08 for 6
c £2.45 for 1 litre
d Same value

2 a Large jar as more g per £
b 75 ml tube as more ml per £
c Large box as more g per £
d 400 ml bottle as more ml per £

3 a £5.11
b Large tin (small £5.11/l, medium £4.80/l, large £4.47/l)

4 a 95p **b** Family size

5 Mary

6 Kelly

7 12-pack 360 ÷ 12 = 30p per sachet. 40-pack 1150 ÷ 40 = 28.75p per sachet. 4 sachets cost 4 × 35 = £1.40 but you get 5, so 140 ÷ 5 = 28p per sachet, so the offer is the best value.

8 a Abe uses 10 × 0.75 = 7.5 litres to do 100 km. Caryl uses 100 ÷ 14 = 7.14 litres to do 100 km and Des uses 100 ÷ (55 × 1.6) × 4.55 = 5.17 litres to do 100 km, so Des's car is the most ecomonical.
b It does not give a 'unit' value, ie miles per gallon or litres per mile.

Exercise 5F

1 a £260 **b** £307.50 **c** £323.75 **d** £289

2 a £7.50 **b** £9.05 **c** £5.80 **d** £10.75

3 **a** 38 h **b** $41\frac{1}{2}$ h **c** 35 h **d** 40 h

4 **a** Fewer hours **b** More pay

5 **a** £540 **b** £702

6 £6.90

7 375 – 330 = 45, 45 ÷ 6 = £7.50. (375 – 12 × 7.50) ÷ 7.50 = 38 hours

8 £$1\frac{1}{4}x$

9 Pay is £442.50 tax is £88.50, NI is 442.50 – 88.50 – 327.45 = 26.55, 26.55 ÷ 442.5 = 0.06, so the NI rate is 6%

10 407 factorises to 1 × 407 or 11 × 37, so Jeff works 37 hours a week at £11 per hour.

Exercise 5G

1 18 mph

2 280 miles

3 52.5 mph

4 11:50 am

5 500 seconds

6 **a** 75 mph **b** 6.5 h
c 175 miles **d** 240 km
e 64 km/h **f** 325 km
g 4.3 h (4 h 18 min)

7 **a** 2.25 h **b** 99 miles

8 **a** 1.25 h **b** 1 h 15 min

9 **a** Sheffield to London via Midland mainline 74.38 mph. Sheffield to London via East Coast mainline 78.26 mph, including the wait at Doncaster
b Doncaster to London 94.12 mph, including the wait at Doncaster

10 **a** 120 km **b** 48 km/h

11 **a** 30 min **b** 6 mph

12 **a** 10 m/s **b** 3.3 m/s **c** 16.7 m/s

13 **a** 90 km/h **b** 43.2 km/h **c** 1.8 km/h

14 18 m/s is 64.8 km/h. 40 km at 64.8 km/h is 0.617 hours ≈ 37 minutes so train arrives at 8.07 am

15

Time	10	10.15	10.30	10.45	11
Ajeet	16	20	24	28	32
Bijay	0	6	12	18	24
Time	11.15	11.30	11.45	12	12.15
Ajeet	36	40	44	48	52
Bijay	30	36	42	48	54

Bijay catches Ajeet at 12 noon

16 Rebecca: 10 minutes at 50 mph covers 8.333 miles, 10 minutes at 70 mph covers 11.666 miles, so total distance is 20 miles in 20 minutes which is 60 mph, so Rebecca is correct.
Nick: 10 miles at 40 mph takes 15 minutes, 10 miles at 60 mph takes 10 minutes, so total distance is 20 miles in 25 minutes, which is 48 mph, so Nick is wrong.

17 Josh should take 40 minutes. Nell should take 50 ÷ 70 × 60 = 43 minutes, but Josh is likely to meet traffic through town so is unlikely to travel at anywhere near 30 mph. Nell is likely to be able to travel at 70 mph on the motorway.

Exercise 5H

1 **a** 0.75 g/cm^3

2 4 pa

3 $8\frac{1}{3}$ g/cm3

4 2 ½ N

5 32 g

6 5 m^2

7 120 cm^3

8 156.8 g

9 30 × 20

10 By the handle as smaller area

11 So they can walk on sand easier due to less pressure on the surface.

12 **a** 19.3 kg
b 19.3 kg. Mass is same
c On largest face 9560 Pa, On smallest face 38600 Pa

13 First statue is the fake as density is approximately 26 g/cm^3

14 Second piece by 1 cm^3

15 0.339 m^3

16 Areas are ½ m^2. 0.8 m^2. 0.4m^2. Sides are 1 m, $\frac{1}{2}$ m and 0.8 m

17 **a** T **b** F **c** F **d** T

Exercise 5I

1 **a** £400 **b** £112.50
c £12.80 **d** £499.46

2 **a** 8 years **b** 12 years

3 **a** **i** 10.5 g **ii** 11.03 g
iii 12.16 g **iv** 14.07 g
b 9 days

4 12 years

5 **a** £14 272.27 **b** 20 years

6 **a** **i** 2550 **ii** 2168 **iii** 1331
b 7 years

7 **a** £6800 **b** £5440 **c** £3481.60

8 **a** **i** 1.9 million litres
ii 1.6 million litres
iii 1.2 million litres
b 10th August

9 **a** **i** 51 980
ii 84 752
iii 138 186
b 2021

10 **a** 21 years **b** 21 years

11 3 years

12 30 years

13 1.1 × 1.1 = 1.21 (21% increase)

14 Bradley Bank account is worth £1032, Monastery Building Society account is worth £1031.30, so Bradley Bank by 70p.

15 4 months: fish weighs $3 \times 1.1^4 = 4.3923$ kg; crab weighs $6 \times 0.9^4 = 3.9366$ kg

16 4 weeks

Exercise 5J

1 **a** 800 g **b** 250 m **c** 60 cm
d £3075 **e** £200 **f** £400

2 80

3 T-shirt £8.40, Tights £1.20, Shorts £5.20, Sweater £10.75, Trainers £24.80, Boots £32.40

4 £833.33

5 £300

6 240

7 £350

8 4750 blue bottles

9 £26.40

10 **a** £1600
b With 10% cut each year he earns £1440 × 12 + £1296 × 12 = £17 280 + £15 552 = £32 832 With immediate 14% cut he earns £1376 × 24 = £33 024, so correct decision

11 **a** 30% **b** 15%

12 Less by $\frac{1}{4}$%

13 £900

14 Calculate the pre-VAT price for certain amounts, and $\frac{5}{6}$ of that amount. Show the error grows as the amount increases. Up to £281 the error is less than £5.

15 £1250

16 £1250

17 0.28 × 5400 = 1512. 1512 × 2.5 = 3780, 3780 ÷ 0.72 = 5250, so population has declined by 150 people.

18 Baz has assumed that 291.2 is 100% instead of 112%. He rounded his wrong answer to the correct answer of £260.

19 35% = $\frac{35}{100}$ which cancels to $\frac{7}{20}$, so the smallest number that could have been surveyed is 20.

Review questions

1 48 mph

2 Definite, as his average speed was 80 miles per hour which is 128 km/h

3 Totals are 40 and 60 giving 2 : 3 and a total of 100. 9 : 11 is a ratio of 45 : 55 so swap 10 and 15

4 **a** 1.73 ÷ 0.04 = 43.25 so 43 horses, 2.64 ÷ 0.065 = 40.62 so 40 cattle and 0.95 ÷ 0.01 = 95. Total 43 + 40 + 95 = 178 animals.
b Horses in field A = 43, Sheep in field B = 2.64 ÷ 0.01 = 264, Cattle in field C, 0.95 ÷ 0.065 = 14.62, so 14 cattle. Total 43 + 264 + 14 = 321 animals

5 100°

6 **a** 22.5 kg **b** 30 kg **c** £19.80

7 £8357.35

8 £375

9 **a** £4945.97
b 5, yes he has £1357.68 in the account so he has rounded to the nearest £10

10 13.04%

11 90

12 Joe pays 41.4 – 4.4 = £37, Lucy pays 41.4 ÷ 1.15 = £36, so Joe's meal cost more.

13 680.4 ÷ 4500 = £0.1512 per units in 2015. 0.1512 ÷ 1.08 = £0.14 pence per unit in 2014. 5400 × £0.14 = £756, so she paid more for the units in 2014.

14 50.50 ÷ 0.9 ÷ 0.85 = 66.01, so price was £66 and other prices are rounded off.

Chapter 6 – Geometry and measures: Angles

Exercise 6A

1 **a** 108° **b** 52° **c** 59°

2 **a** 57° **b** 40°

3 No; 45° + 125° = 170° and for a straight line it should be 180°.

4 **a** $x = 100°$ **b** $x = 110°$ **c** $x = 30°$

5 **a** $x = 55°$ **b** $x = 45°$ **c** $x = 12.5°$

6 **a** $x = 34°$, $y = 98°$ **b** $x = 70°$, $y = 120°$
c $x = 20°$, $y = 80°$

7 6 × 60° = 360°; imagine six of the triangles meeting at a point

8 $x = 35°$, $y = 75°$; $2x = 70°$ (opposite angles), so $x = 35°$ and $x + y = 110°$ (angles on a line), so $y = 75°$

Exercise 6B

1 **a-c** Students' own drawings **d** 180°

2 **a** 60° **b** Equilateral triangle
c Same length

3 **a** 70° each **b** Isosceles triangle
c Same length

4 **a** 109° **b** 130° **c** 135°

5 65°

6 Joe is not correct as DFE = 30, DEF = 75 hence angle D = 180 – 105 = 75 but Hannah is correct as FED = FDE = 75°

7 $a = 35°$ (angles in a triangle) because the other angles in the triangle are 65° (angles on a line) and 80° (opposite angles) giving a total of 145, this subtracted from the 180 degrees in a triangle leaves the answer of 35

8 Missing angle = y, $x + y = 180°$ and $a + b + y = 180°$ so $x = a + b$

9 32

10 72°

Exercise 6C

1 2, 2, 360°

2 3, 3, 540°

3 4, 4, 720°

4

Shape	Number of sides	Triangles	Angle sum
Triangle	3	1	180
Quadrilateral	4	2	360
Pentagon	5	3	540
Hexagon	6	4	720
Heptagon	7	5	900
Octagon	8	6	1080
Nonagon	9	7	1260
Decagon	10	8	1440

5 18, 18, 3240

Exercise 6D

1 **a** 90° **b** 150° **c** 80°

2 **a** No, total is 350° **b** Yes, total is 360°
c No, total is 350° **d** No, total is 370°
e Yes, total is 360° **f** Yes, total is 360°

3 **a** 90° **b** Rectangle **c** Square

4 **a** 120° **b** 136° **c** 149°
d 126° **e** 114°

5 60° + 60° + 120° + 120° + 120° + 240° = 720°

6 $y = 360° – 4x$; $2x + y + 2x = 360°$, $4x + y = 360°$, so $y = 360° – 4x$

7 $x = 40°$, so the smaller angle is 60°

Exercise 6E

1

Shape	Number of sides	Interior angle sum	Each interior angle
octagon	8	1080	135
nonagon	9	1260	140
decagon	10	1440	144

2

Regular polygon	Number of sides	Interior angle	Exterior angle
square	4	90	90
pentagon	5	108	72
hexagon	6	120	60
octagon	8	135	45
nonagon	9	140	40
decagon	10	144	36

3 **a** **i** 45° **ii** 8 **iii** 1080°
b **i** 20° **ii** 18 **iii** 2880°
c **i** 15° **ii** 24 **iii** 3960°
d **i** 36° **ii** 10 **iii** 1440°

4 **a** **i** 172° **ii** 45 **iii** 7740°
b **i** 174° **ii** 60 **iii** 10 440°
c **i** 156° **ii** 15 **iii** 2340°
d **i** 177° **ii** 120 **iii** 21 240°

5 **a** Exterior angle is 7°, which does not divide exactly into 360°
b Exterior angle is 19°, which does not divide exactly into 360°
c Exterior angle is 11°, which does divide exactly into 360°
d Exterior angle is 70°, which does not divide exactly into 360°

6 **a** 7° does not divide exactly into 360°
b 26° does not divide exactly into 360°
c 44° does not divide exactly into 360°
d 13° does not divide exactly into 360°

7 48°; $\frac{1440° - 5 \times 240°}{5}$

8 10

9 $x = 45°$, they are the same, true for all regular polygons

10 Three are 135°and two are 67.5°

11 72°, 72°, 108°, 144°, 144°

12 93° or 273°

Exercise 6F

1 **a** *d* **b** *f* **c** *d* **d** *f* **e** *f* **f** *e*

2 **a** $b = c = 70°$ **b** $d = 75°$, $e = f = 105°$
c $g = 50°$, $h = i = 130°$ **d** $n = m = 80°$
e $g = i = 65°$, $h = 115°$ **f** $j = k = 72°$, $l = 108°$

3 **a** $a = 95°$ **b** $b = 66°$, $c = 114°$

4 **a** $x = 30°$, $y = 120°$ **b** $x = 25°$, $y = 105°$
c $x = 30°$, $y = 100°$ **d** $x = 50°$, $y = 110°$
e $x = 25°$, $y = 55°$ **f** $x = 20°$, $y = 140°$

5 290°; x is double the angle allied to 35°, so is $2 \times 145°$

6 angle BDC = 66° (angles in a triangle = 180°) angle BDE = 114° (angles on a line = 180°) so a = 66° (corresponding angle or allied angle)

7 Angle PQD = 64° (alternate angles), so angle DQY = 116° (angles on a line = 180°)

8 Use alternate angles to see b, a and c are all angles on a straight line, and so total 180°

9 Third angle in triangle equals q (alternate angle), angle sum of triangle is 180°.

Exercise 6G

1 **a** $a = 110°$, $b = 55°$ **b** $g = i = 63°$, $h = 117°$
c e = f = 94°

2 **a** $a = 58°$, $b = 47°$ **b** $c = 141°$, $d = 37°$
c e = g = 65°, $f = 115°$

3 **a** 65° **b** 60° **c** 68°

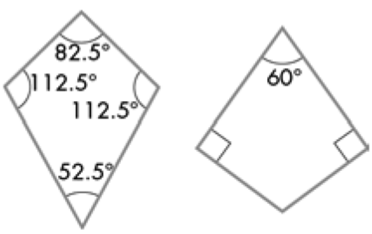
4 both 129°

5 Marie is correct, a rectangle is a parallelogram with all angles equal to 90°

6 **a** 65°
b Trapezium, angle A + angle D = 180° and angle B + angle C = 180°

7 135

8



9 A trapezium; angles add up to $10x$, two angles x and $4x = 2x + 3x$, the other pair of angles. Hence each pair adds up to 180 (since 2 × 180 = 360). Hence two pairs of allied angles, hence a trapezium. Alternatively you could have found that x = 36 which will give the same result.

Exercise 6H

1 **a** Student's scale drawing.
b About 19 m so about 38 plants

2 **a** **i** 65 km **ii** 212 km
iii 114 km **iv** 36 km
b **i** 83 km **ii** 196 km
iii 149 km **iv** 130 km

3 **a** 36 km **b** 2 000 000

4 1 : 63 360

5 **a** 110°, 12.6 km **b** 250°, 4.5 km
c 091°, 11.8 km **d** 270°, 8.4 km
e 130°, 7.2 km **f** 180°, 4.2 km

6 Students' Sketches

7 **a** Sketch
b D is due south of B and B is east of A, so A must be west of D. A bearing to the west will be greater than 180°

8 **a** 090°, 180°, 270° **b** 000°, 270°, 180°

9 **a** 045° **b** 286°
c measure the distance from X to Y and divide 15 by this to find the scale of the map. Then measure the distance from Y to P and multiply by the scale factor

10 **a** 250° **b** 325° **c** 144°

11 **a** 900 m **b** 280°
c angle NHS = 150° and HS = 3 cm

12 108°

13 255°

14 9.92 km

Review questions

1 16

2 **a** **i** 115°
ii Vertically opposite angles are equal and co-interior angles add up to 180°
b the angles do not add up to 360°

3 **a** 50° **b** 32.5°

4 A five sided shape can be split into 3 triangles hence 3 × 180
= 540°

5 150°

6 angle TQP = 37°(alternate angles), PTQ = 180 – (29 + 37) = 114° (angles in a triangle), QTS = 180 – 114 (angles on a line) = 66°

7 333°

8 360 ÷ 8 = 45°; exterior angle formula is 360 ÷ number of sides, in this case 8

9 180 – (360 ÷ 6) = 120° or (180 × 4) ÷ 6 = 120°

10 Selvi might be correct. You will need to draw one example showing this is not a kite, and one example showing that this could be a kite

11 **a** Student’s own sketch **b** 12.4 km

Chapter 7 – Geometry and measures: Transformations, constructions and loci

Exercise 7A

1 **a** SAS **b** SSS **c** ASA
d RHS **e** SSS **f** ASA

2 **a** SSS. A to R, B to P, C to Q
b SAS. A to R, B to Q, C to P

3 **a** 60° **b** 80° **c** 40° **d** 5 cm

4 **a** 110° **b** 55° **c** 85° **d** 110° **e** 4 cm

5 SSS or RHS

6 SSS or SAS or RHS

7 For example, use ∠ADE and ∠CDG. AD = CD (sides of large square), DE = DG (sides of small square), ∠ADE = ∠CDG (angles sum to 90° with ∠ADG), so ∆ADE ≡ ∆CDG (SAS), so AE = CG

8 AB and PQ are the corresponding sides to the 42° angle, but they are not equal in length.

Exercise 7B

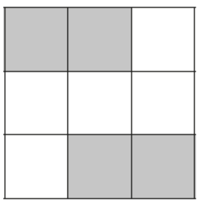
1 **a** 4 **b** 5 **c** 6 **d** 4 **e** 6

2 **a** 2 **b** 2 **c** 2 **d** 2 **e** 2

3 A, B, C, D, E, F, G, J, K, L, M, P, Q, R, T, U, V, W, Y

4 **a** 1
b the central white star or the large dark green star
c order 16 – the light green star around the central white star, or order 9 – the light green shape between the outer petals and the inner stars

5 for example:



6

		Number of lines of symmetry			
		0	1	2	3
Order of` rotational symmetry	1	D	A		
	2	E		B	
	3				C

7 She is correct since the angle sum around the centre point is 360 and 360 ÷ 3 = 120

8 Yes she is correct. A triangle can only have 1 or 3 lines of symmetry. If a triangle has 3 lines of symmetry it also has rotational symmetry of order 3, so this triangle must only have 1 line of symmetry. This will mean it has two angles identical and two sides, and hence an isosceles triangle.

Exercise 7C

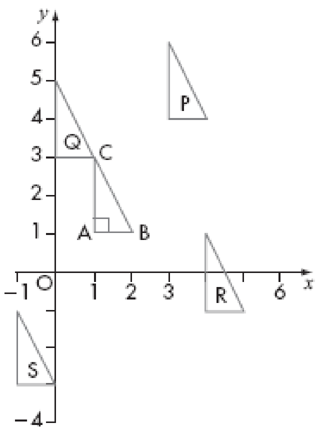
1 a i $\begin{pmatrix}1\\3\end{pmatrix}$ **ii** $\begin{pmatrix}4\\2\end{pmatrix}$ **iii** $\begin{pmatrix}2\\-1\end{pmatrix}$ **iv** $\begin{pmatrix}5\\1\end{pmatrix}$

b i $\begin{pmatrix}-1\\-3\end{pmatrix}$ **ii** $\begin{pmatrix}3\\-1\end{pmatrix}$ **iii** $\begin{pmatrix}-2\\3\end{pmatrix}$ **iv** $\begin{pmatrix}3\\3\end{pmatrix}$

c i $\begin{pmatrix}-2\\-3\end{pmatrix}$ **ii** $\begin{pmatrix}1\\-1\end{pmatrix}$ **iii** $\begin{pmatrix}-5\\4\end{pmatrix}$ **iv** $\begin{pmatrix}0\\4\end{pmatrix}$

d i $\begin{pmatrix}3\\2\end{pmatrix}$ **ii** $\begin{pmatrix}-4\\2\end{pmatrix}$ **iii** $\begin{pmatrix}5\\-4\end{pmatrix}$ **iv** $\begin{pmatrix}-2\\-7\end{pmatrix}$

2



3 **a** $\begin{pmatrix}-3\\-1\end{pmatrix}$ **b** $\begin{pmatrix}4\\-4\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-2\end{pmatrix}$ **d** $\begin{pmatrix}4\\7\end{pmatrix}$

e $\begin{pmatrix}-1\\5\end{pmatrix}$ **f** $\begin{pmatrix}1\\6\end{pmatrix}$ **g** $\begin{pmatrix}-4\\4\end{pmatrix}$ **h** $\begin{pmatrix}-4\\-7\end{pmatrix}$

4 10 × 10 = 100 (including $\begin{pmatrix}0\\0\end{pmatrix}$)

5 **a** Check students’ designs for a Snakes and ladders board.
b because the ladders always mean moving up the board
ii the snakes always mean moving down the board

6 $\begin{pmatrix}-x\\-y\end{pmatrix}$

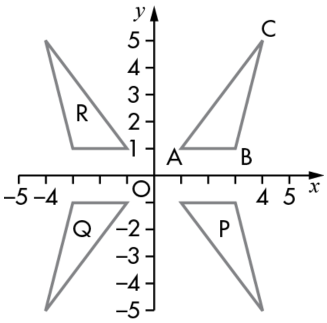
7 **a** Check student's diagram **b** $\begin{pmatrix} -300 \\ -500 \end{pmatrix}$

8 $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

9 Under a translation every points moves with the same vector, hence all the sides are the same length, so we can use the SSS rule of congruency.

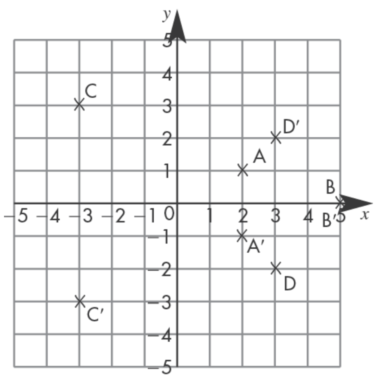
Exercise 7D

1 **a–e**



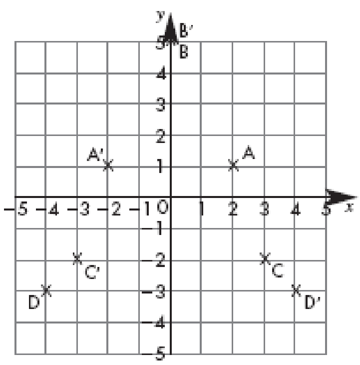
f Reflection in the y-axis

2 **a–b**



c y-value changes sign
d $(a, -b)$

3 **a–b**



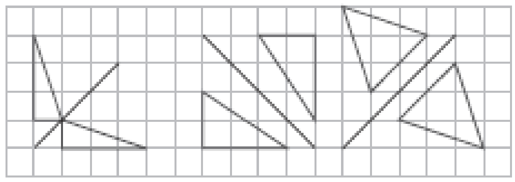
c x-value changes sign
d $(-a, b)$

e Any three points with x co-ordinate 0, e.g. (0, 1), (0, 2), (0, 3)

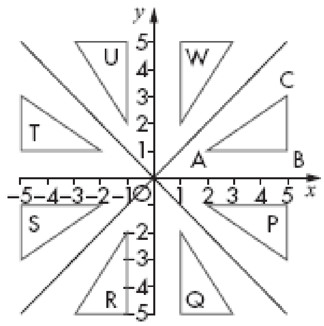
4 Possible answer: Take the centre square as ABCD then reflect this square each time in the line, AB, then BC, then CD and finally AD.

5 $x = -1$

6

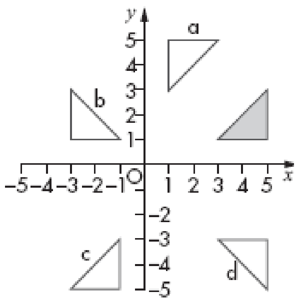


7 **a–i**

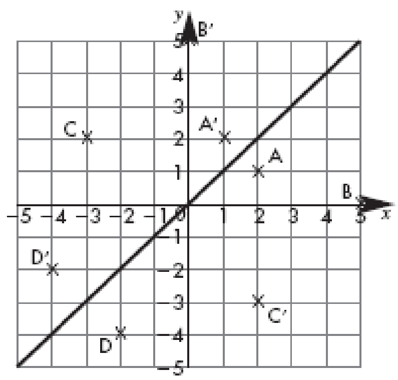


j A reflection in $y = x$

8



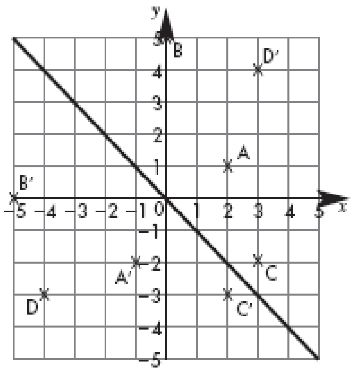
9 **a–c**



d Coordinates are reversed: x becomes y and y becomes x

e (b, a)
f Any point with x and y co-ordinates the same, e.g. (1, 1), (2, 2)

10 a–c

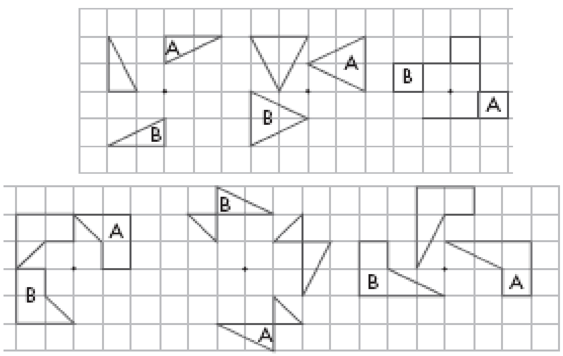


d Coordinates are reversed and change sign, x becomes $-y$ and y becomes $-x$
e ($-b$, $-a$)

11 Because a reflection is exactly the same shape as the original, just in a different orientation, hence we can use the rule SSS to show the two shapes are congruent.

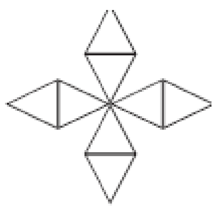
Exercise 7E

1 a



b i Rotation 90°anticlockwise
ii Rotation 180°

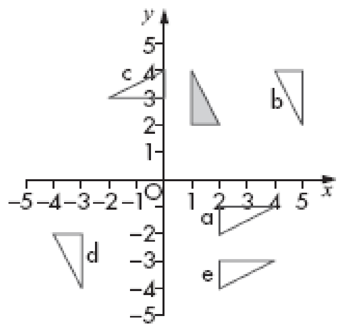
2 a



b rotate shape 60° around point B, then repeat another four times.

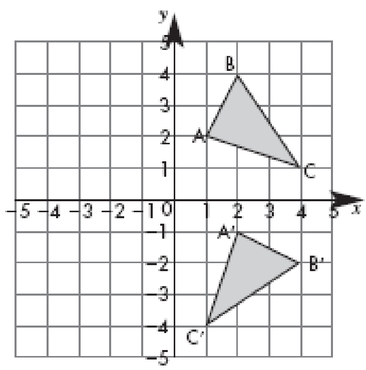
3 Possible answer: If ABCD is the centre square, rotate about A 90° anticlockwise, rotate about new B 180°, now rotate about new C 180°, and finally rotate about new D 180°.

4



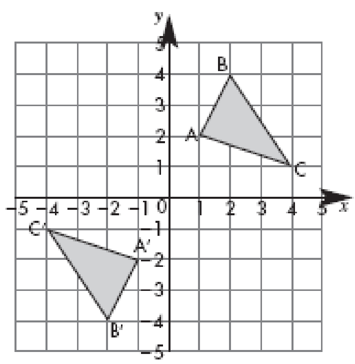
5 a 90° anticlockwise
b 270° anticlockwise
c 300° clockwise
d 260° clockwise

6 a b c i



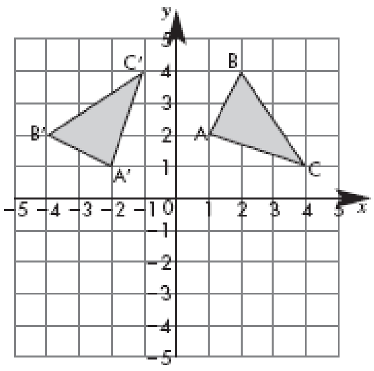
ii A' (2, –1), B' (4, –2), C' (1, –4)
iii Original coordinates (x, y) become (y, $-x$)
iv Yes

7 i



ii A' (–1, –2), B' (–2, –4), C' (–4, –1)
iii Original coordinates (x, y) become ($-x$, $-y$)
iv Yes

8 **i**



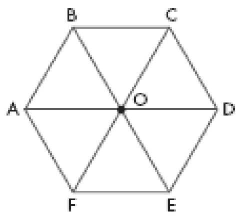
ii A' (–2, 1), B' (–4, 2), C' (–1, 4)
iii Original coordinates (x, y) become ($-y$, x)
iv Yes

9 The centre of rotation

10 Show by drawing a shape or use the fact that (a, b) becomes (a, $-b$) after reflection in the x-axis, and (a, $-b$) becomes ($-a$, $-b$) after reflection in the y-axis, which is equivalent to a single rotation of 180°.

11 she is correct

12 **a**



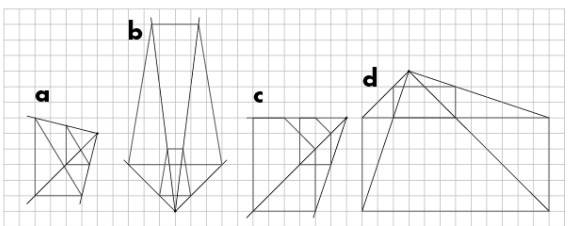
b i Rotation 60°clockwise about O
ii Rotation 120°clockwise about O
iii Rotation 180°about O
iv Rotation 240°clockwise about O
c i Rotation 60°clockwise about O
ii Rotation 180°about O

13 Rotation 90° anticlockwise about (3, –2)

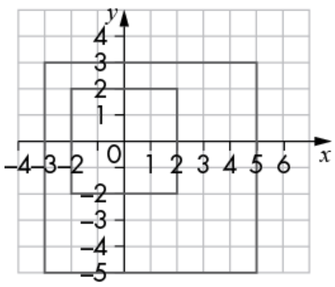
14 Because under a rotation, the lengths of the original shape are preserved, so we can use the rule SSS to show they are congruent.

Exercise 7F

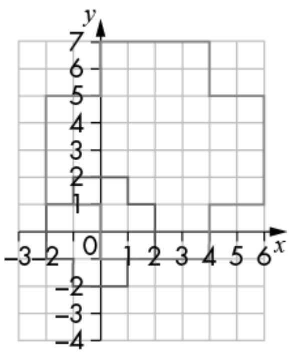
1



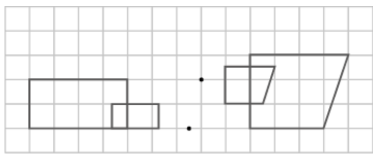
2 **a**



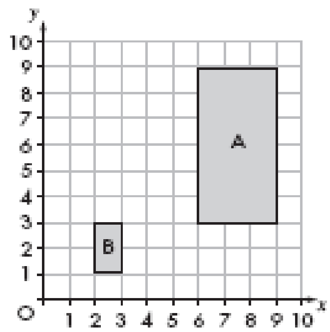
b



3



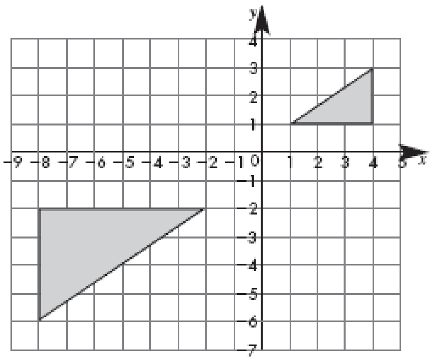
4 **a**



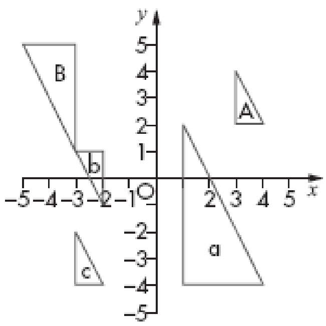
b 3 : 1
c 3 : 1
d 9 : 1

5 **a** (1, 1), (3, –3), (–5, –5) **b** (1, 1)

6



7 a–c



- **d** Scale factor $-\frac{1}{2}$, centre (1, 3)
- **e** Scale factor –2, centre (1, 3)
- **f** Scale factor –1, centre (–2.5, –1.5)
- **g** Scale factor –1, centre (–2.5, –1.5)
- **h** Same centres, and the scale factors are reciprocals of each other

8 Enlargement, scale factor –2, about (1, 3)

9 Because the sides of triangle C are all larger than the original triangle B, and so the SSS rule will not apply.

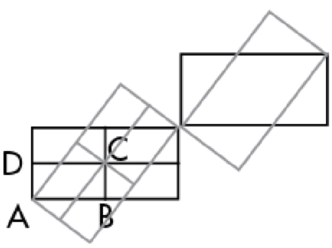
Exercise 7G

1 (–4, –3)

2 **a** (–5, 2)
b Reflection in *y*-axis

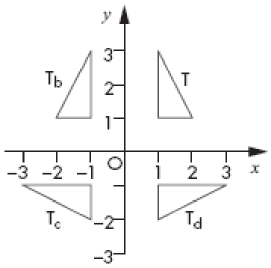
3 A: translation $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, B: reflection in *y*-axis, C: rotation 90°clockwise about (0, 0), D: reflection in $x = 3$, E: reflection in $y = 4$, F: enlargement by scale factor 2, centre (0, 1)

4



5. **a** T_1 to T_2: rotation 90°clockwise about (0, 0)
b T_1 to T_6: rotation 90°anticlockwise about (0, 0)
c T_2 to T_3: translation $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$
d T_6 to T_2: rotation 180°about (0, 0)
e T_6 to T_5: reflection in *y*-axis
f T_5 to T_4: translation $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

6 **a–d**



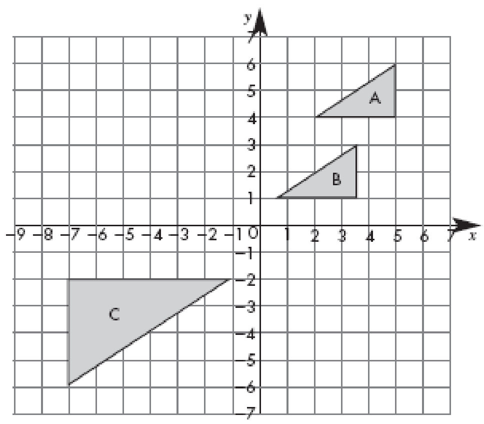
e T_d to T: rotation 90° anticlockwise about (0, 0)

7 (3, 1)

8 Reflection in *x*-axis, translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$, rotation 90°clockwise about (0, 0)

9 Translation $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$, reflection in *x*-axis, rotation 90°clockwise about (0, 0)

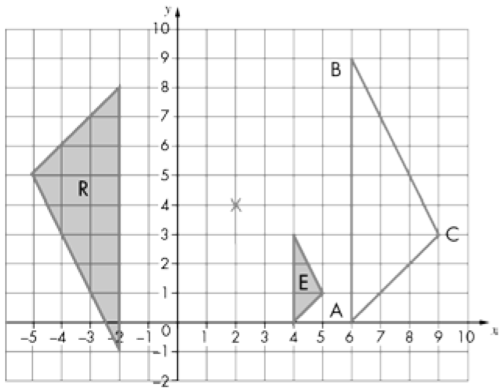
10 **a**



b enlargement of scale factor $-\frac{1}{2}$ about (1, 2)

11 No, this can be shown with an example.

12 a-b



c An enlargement, scale factor –3, centre (2.5, 2)

13 (10, 10)

Exercise 7H

1–9 Practical work; check students' constructions

10 The centre of the circle

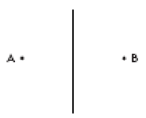
11 Start with a base line AB; then construct a perpendicular to the line from point A. At point B, construct an angle of 60°. Ensure that the line for this 60° angle crosses the perpendicular line; where they meet will be the final point C.

12–14 Practical work; check students' constructions

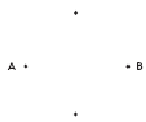
Exercise 7I

1 Circle with radius:
a 2 cm **b** 4 cm **c** 5 cm

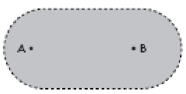
2 **a**



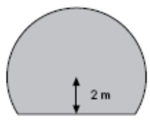
b



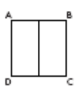
c



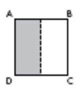
3 **a** Circle with radius 4 m **b**



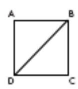
4 **a**



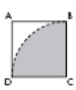
b



c

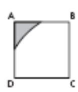


d

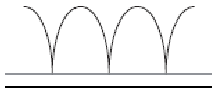


e

f

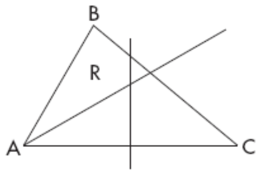


5

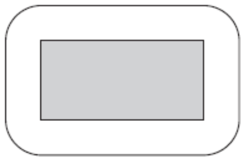


6

7 Construct the bisector of angle BAC and the perpendicular bisector of the line AC.



8

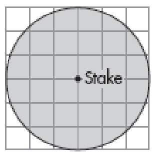


9. Start with a base line, AB, 3 cm long. At point A, draw a few points all 3 cm away from A towards the upper right side. Lightly join these dots with an arc. You can now find the point that is 3 cm away from point B and draw the equilateral triangle.

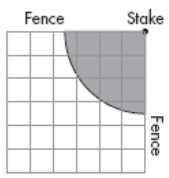
10. Gary is correct about the triangle inside, but not a triangle outside as there will be three straight lines, parallel to each side of the triangle, then these straight lines will be joined with arcs centred on the vertices of the original triangle.

Exercise 7J

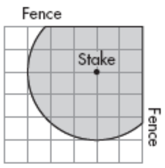
1



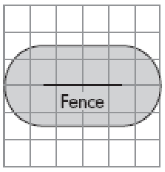
2 **a**



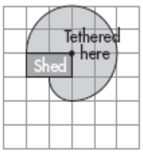
b



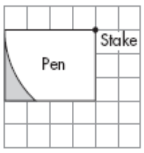
3



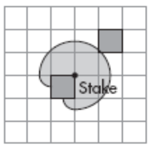
4



5



6



7 No, if you accurately draw a diagram showing the path of each boat, you will find the boat leaving from point B meets the path of the other boat in a much shorter time as it's a smaller distance than fom A to the cross over point.)

8 On a map, draw a straight line from Newcastle to Bristol, construct the line bisector, then the search will be anywhere on the sea along that line.

9 a Sketch should show a circle of radius 6 cm around London and one of radius 4 cm around Glasgow.

b No

c Yes

10 a Yes

b Sketch should show a circle of radius 4 cm around Leeds and one of radius 4 cm around Exeter. The area where they overlap should be shaded.

11 a This is the perpendicular bisector of the line from York to Birmingham. It should pass just below Manchester and just through the top of Norwich.

b Sketch should show a circle of radius 7 cm around Glasgow and one of radius 5 cm around London. The area where they overlap should be shaded.

c The transmitter can be built anywhere on line constructed in part **a** that is within the area shown in part **b**.

12 Sketch should show two circles around Birmingham, one of radius 3 cm and one of radius 5 cm. The area of good reception is the area between the two circles.

13 Sketch should show a circle of radius 6 cm around Glasgow, two circles around York, one of radius 4 cm and one of radius 6 cm and a circle around London of radius 8 cm. The small area in the Irish Sea that is between the two circles around York and inside both the circle around Glasgow and the circle around London is where the boat can be.

14 Sketch should show two circles around Newcastle upon Tyne, one of radius 4 cm and one of radius 6 cm, and two circles around Bristol, one of radius 3 cm and one of radius 5 cm. The area that is between both pairs of circles is the area that should be shaded.

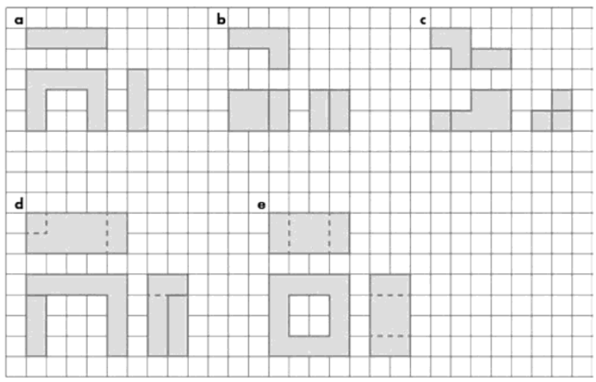
15 Sketch should show the perpendicular bisector of the line running from Newcastle upon Tyne to Manchester and that of the line running from Sheffield to Norwich. Where the lines cross is where the oil rig is located.

16 Sketch should show the perpendicular bisector of the line running from Glasgow to Norwich and that of the line running from Norwich to Exeter. Where the lines cross is where Fred's house is.

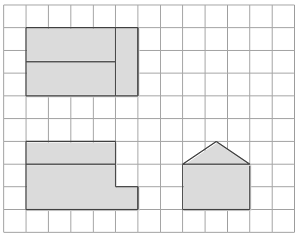
17 Leeds

Exercise 7K

1

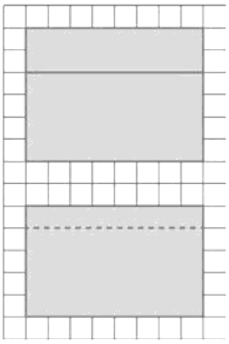


2



3 **a** A cylinder
b A hexagonal prism

4



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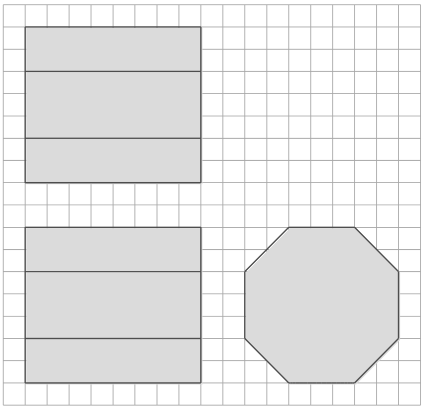

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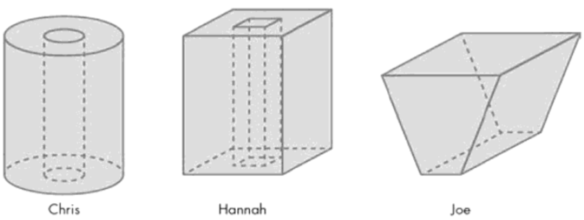
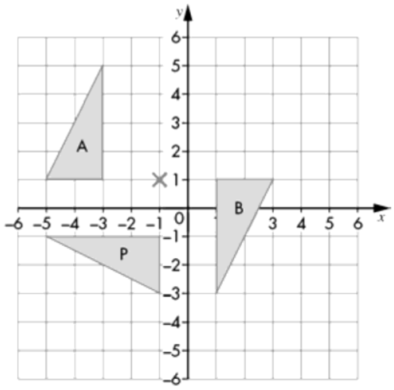


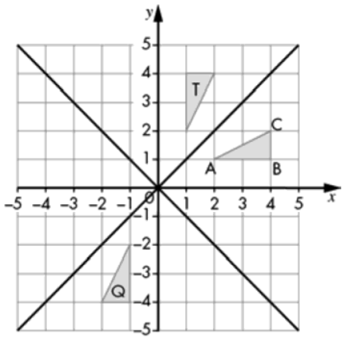
Review questions

1 **a-b**



c rotation of 180° about (–1, 1)
d (–1, 1)

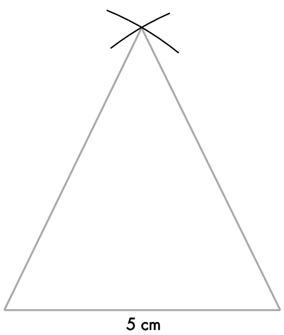
2 **a-b**



c rotation of 180° about O

3 You should have measured the error of the angle size and converted that to a percentage error.

4



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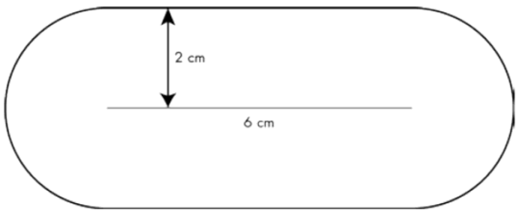
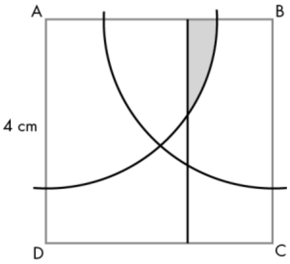
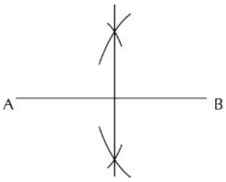


6

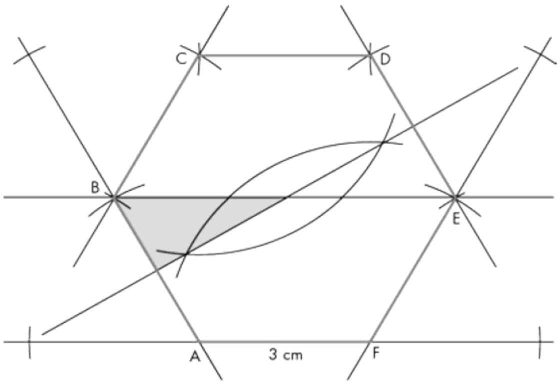


7 **a**



b … an equal distance from A and B

8



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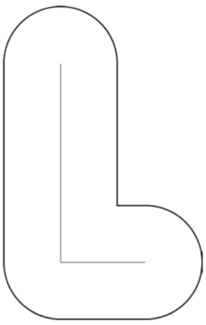
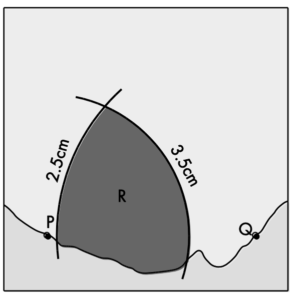
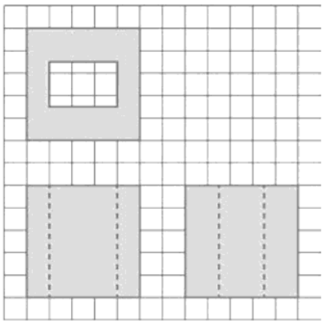


10



11 A (9, 0) B (11, –3) C (2, –1)

12



13 It is not always true

14 In a rhombus all sides are the same length, so AB = BC = AD = DC, AC is a common length in both triangles, so each triangle has the three sides matching, SSS.

Chapter 8 – Algebra: Algebraic manipulation

Exercise 8A

1 **a** 13 **b** −3 **c** 5

2 **a** 1.4 **b** 1.4 **c** −0.4

3 **a** 13 **b** 74 **c** 17

4 **a** 75 **b** 22.5 **c** −135

5 **a** 2.5 **b** −20 **c** 2.5

6 **a** $\frac{150}{n}$ **b** £925

7 **a** 2 × 8 + 6 × 11 − 3 × 2 = 76
b 5 × 2 − 2 × 11 + 3 × 8 = 12

8 **a** One odd one even value, different from each other.
b Any valid combination, e.g. $x = 1$, $y = 2$

9 **a** **i** Odd **ii** Odd
iii Even **iv** Odd
b Any valid expression such as $xy + z$

10 **a** £20
b **i** −£40
ii Delivery cost will be zero.
c 40 miles

11 A expression, B formula, C identity, D equation

12 **a** First term is cost of petrol, each mile is a tenth of £0.98. Second term is the hire cost divided by the miles.
b 29.8p per mile

Exercise 8B

1 **a** $6 + 2m$ **b** $10 + 5l$
c $12 - 3y$ **d** $20 + 8k$
e $6 - 12f$ **f** $10 - 6w$
g $10k + 15m$ **h** $12d - 8n$
i $t^2 + 3t$ **j** $k^2 - 3k$
k $4t^2 - 4t$ **l** $8k - 2k^2$
m $8g^2 + 20g$ **n** $15h^2 - 10h$
o $y^3 + 5y$ **p** $h^4 + 7h$
q $k^3 - 5k$ **r** $3t^3 + 12t$
s $15d^3 - 3d^4$ **t** $6w^3 + 3tw$
u $15a^3 - 10ab$ **v** $12p^4 - 15mp$
w $12h^3 + 8h^2g$ **x** $8m^3 + 2m^4$

2 **a** $5(t - 1)$ and $5t - 5$
b Yes, as $5(t - 1)$ when $t = 4.50$ is $5 \times 3.50 =$ £17.50

3 He has worked out 3 × 5 as 8 instead of 15 and he has not multiplied the second term by 3. Answer should be $15x - 12$.

4 **a** $3(2y + 3)$
b $2(6z + 4)$ or $4(3z + 2)$

5 **a** $22 + 5t$ **b** $21 + 19k$
c $22 + 2f$ **d** $14 + 3g$

6 **a** $2 + 2h$ **b** $9g + 5$
c $17k + 16$ **d** $6e + 20$

7 **a** $4m + 3p + 2mp$
b $3k + 4h + 5hk$
c $12r + 24p + 13pr$
d $19km + 20k - 6m$

8 **a** $9t^2 + 13t$ **b** $13y^2 + 5y$
c $10e^2 - 6e$ **d** $14k^2 - 3kp$

9 **a** $17ab + 12ac + 6bc$
b 18wy + 6ty − 8tw
c 14mn − 15mp − 6np
d $8r^3 - 6r^2$

10 **a** $5(f + 2s) + 2(2f + 3s) = 9f + 16s$
b £$(270f + 480s)$
c £42 450 − £30 000 = £12 450

11 For x-coefficients, 3 and 1 or 1 and 4; for y-coefficients, 5 and 1 or 3 and 4 or 1 and 7

12 $5(3x + 2) - 3(2x - 1) = 9x + 13$

Exercise 8C

1 **a** $6(m + 2t)$ **b** $3(3t + p)$
c $4(2m + 3k)$ **d** $4(r + 2t)$
e $m(n + 3)$ **f** $g(5g + 3)$
g $2(2w - 3t)$ **h** $y(3y + 2)$
i $t(4t - 3)$ **j** $3m(m - p)$
k $3p(2p + 3t)$ **l** $2p(4t + 3m)$
m $4b(2a - c)$ **n** $5bc(b - 2)$
o $2b(4ac + 3de)$ **p** $2(2a^2 + 3a + 4)$
q $3b(2a + 3c + d)$ **r** $t(5t + 4 + a)$
s $3mt(2t - 1 + 3m)$ **t** $2ab(4b + 1 - 2a)$
u $5pt(2t + 3 + p)$

2 **a** Mary has taken out a common factor.
b Because the bracket adds up to £10.
c £30

3 **a**, **d**, **f** and **h** do not factorise.
b $m(5 + 2p)$ **c** $t(t - 7)$ **e** $2m(2m - 3p)$
g $a(4a - 5b)$ **i** $b(5a - 3bc)$

4 **a** Bernice
b Aidan has not taken out the largest possible common factor. Craig has taken m out of both terms but there isn't an m in the second term.

5 There are no common factors.

6 Perimeter $= 2x + 8 + x + 5 + 5x + 4 + 9x - 3 + 10 - x = 16x + 24 = 8(2x + 3)$

7 $\dfrac{4x^2 - 12x}{2x - 6}$

Exercise 8D

1 **a** $x^2 + 5x + 6$ **b** $t^2 + 7t + 12$
c $w^2 + 4w + 3$ **d** $m^2 + 6m + 5$

2 **a** $p^2 + 3p - 70$ **b** $u^2 - 12u + 32$
c $k^2 + 2k - 15$ **d** $z^2 - 12z + 27$

3 **a** should be 35 on the end
b should be – 80
c should be $-10x$
d should be $12y$
e should be $-9z$

Exercise 8E

1 **a** $k^2 + 8k + 15$ **b** $a^2 + 5a + 4$
c $x^2 + 2x - 8$ **d** $t^2 + 2t - 15$
e $w^2 + 2w - 3$ **f** $f^2 - f - 6$

2 **a** $r^2 - 10r + 16$ **b** $s^2 - 17s + 70$
c $d^2 - 17d + 16$ **d** $m^2 - 9m + 18$
e $q^2 - 20q + 99$ **f** $y^2 - 13y + 40$

3 **a** $20a$ **b** $3b$ **c** 200
d –11d **e**12e, 28

Exercise 8F

1 **a** $g^2 - 3g - 4$ **b** $y^2 + y - 12$
c $x^2 + x - 12$ **d** $p^2 - p - 2$
e $k^2 - 2k - 8$ **f** $y^2 + 3y - 10$
g $a^2 + 2a - 3$

2 **a** $x^2 - 9$ **b** $t^2 - 25$ **c** $m^2 - 16$
d $t^2 - 4$ **e** $y^2 - 64$ **f** $p^2 - 1$
g $25 - x^2$ **h** $49 - g^2$ **i** $x^2 - 36$

3 $(x + 2)$ and $(x + 3)$

4 **a** B: $1 \times (x - 2)$
C: 1×2
D: $2 \times (x - 1)$
b $(x - 2) + 2 + 2(x - 1) = 3x - 2$
c Area A $= (x - 1)(x - 2) =$ area of square minus areas (B + C + D) $= x^2 - (3x - 2) = x^2 - 3x + 2$

5 **a** $x^2 - 9$
b **i** 9991 **ii** 39 991

6 **a** $y^2 + 14y + 45$
b **i** 45.1401 **ii** 45.4209
iii 44.7204 **iv** 11 445

7 **a** $x^2 + 2x + 1$
b $x^2 – 2x + 1$
c $x^2 – 1$
d Expand the expressions $[(x + 1) + (x – 1)]^2$ and $(x + 1)^2 + 2(x + 1)(x – 1) + (x – 1)^2$ to show that they are equal.

Exercise 8G

1 **a** $6x^2 + 11x + 3$ **b** $12y^2 + 17y + 6$
c $6t^2 + 17t + 5$ **d** $8t^2 + 2t – 3$
e $10m^2 – 11m – 6$ **f** $12k^2 – 11k – 15$
g $6p^2 + 11p – 10$ **h** $10w^2 + 19w + 6$
i $6a^2 – 7a – 3$ **j** $8r^2 – 10r + 3$
k $15g^2 – 16g + 4$ **l** $12d^2 + 5d – 2$
m $8p^2 + 26p + 15$ **n** $6t^2 + 7t + 2$
o $6p^2 + 11p + 4$ **p** $–10t^2 – 7t + 6$
q $–6n^2 + n + 12$ **r** $6f^2 – 5f – 6$
s $–10q^2 + 7q + 12$ **t** $–6p^2 – 7p + 3$
u $–6t^2 + 10t + 4$

2 **a** $(3x – 2)(2x + 1) = 6x^2 – x – 2$
$(2x – 1)(2x – 1) = 4x^2 – 4x + 1$
$(6x – 3)(x + 1) = 6x^2 + 3x – 3$
$(4x + 1)(x – 1) = 4x^2 – 3x – 1$
$(3x + 2)(2x + 1) = 6x^2 + 7x + 2$
b Multiply the x terms to match the x^2 term and/or multiply the constant terms to get the constant term in the answer.

3 **a** $4x^2 – 1$ **b** $9t^2 – 4$ **c** $25y^2 – 9$
d $16m^2 – 9$ **e** $4k^2 – 9$ **f** $16h^2 – 1$
g $4 – 9x^2$ **h** $25 – 4t^2$ **i** $36 – 25y^2$
j $a^2 – b^2$ **k** $9t^2 – k^2$ **l** $4m^2 – 9p^2$
m $25k^2 – g^2$ **n** $a^2b^2 – c^2d^2$ **o** $a^4 – b^4$

4 **a** $a^2 – b^2$
b Dimensions: $a + b$ by $a – b$; Area: $a^2 – b^2$
c Areas are the same, so $a^2 – b^2 = (a + b) \times (a – b)$

5 First shaded area is $(2k)^2 – 1^2 = 4k^2 – 1$
Second shaded area is $(2k + 1)(2k – 1) = 4k^2 – 1$

6 **a** $3w^2 + 22w + 24$
b **i** 32 224 **ii** 23.7803
iii 24.000 440 0012

7 **a** $49a^2 – b^2$ **b** 4896

Exercise 8H

1 **a** $x^2 + 10x + 25$ **b** $m^2 + 8m + 16$
c $t^2 + 12t + 36$ **d** $p^2 + 6p + 9$
e $m^2 – 6m + 9$ **f** $t^2 – 10t + 25$
g $m^2 – 8m + 16$ **h** $k^2 – 14k + 49$

2 **a** $9x^2 + 6x + 1$ **b** $16t^2 + 24t + 9$
c $25y^2 + 20y + 4$ **d** $4m^2 + 12m + 9$
e $16t^2 – 24t + 9$ **f** $9x^2 – 12x + 4$
g $25t^2 – 20t + 4$ **h** $25r^2 – 60r + 36$
i $x^2 + 2xy + y^2$ **j** $m^2 – 2mn + n^2$
k $4t^2 + 4ty + y^2$ **l** $m^2 – 6mn + 9n^2$
m $x^2 + 4x$ **n** $x^2 – 10x$
o $x^2 + 12x$ **p** $x^2 – 4x$

3 **a** Bernice has just squared the first term and the second term. She hasn't written down the brackets twice.
b Pete has written down the brackets twice but has worked out $(3x)^2$ as $3x^2$ and not $9x^2$.
c $9x^2 + 6x + 1$

4 Whole square is $(2x)^2 = 4x^2$.
Three areas are $2x – 1$, $2x – 1$ and 1.
$4x^2 – (2x – 1 + 2x – 1 + 1) = 4x^2 – (4x – 1)$
$= 4x^2 – 4x + 1$

5 **a** $9p^6 + 42p^3q^7 + 49q^{14}$

6 **a** $9k^2 + 24k + 16$
b **i** 16.2409 **ii** 92 416 **iii** 16.120 225

Exercise 8I

1 **a** $x^3 + 6x^2 + 11x + 6$
b $x^3 – 49x – 120$
c $x^3 + 9x^2 – 4x – 36$

2 **a** $x^3 + 7x^2 – 17x + 9$
b $x^3 + x^2 – x – 10$

3 **a** $x^3 + 12x^2 + 48x + 64$
b $x^3 – 18x^2 + 108x – 216$
c $x^3 + 3ax^2 + 3a^2x + a^3$

4 **abc** $x^3 + 11x^2 + 31x + 21$
d Can be performed in any order

5 **a** $x^3 + (a + b + c)x^2 + (ab + ac + bc)x + abc$
b $p = 0$, $q = –19$, $r = –30$

6 **a** $x^3 – 15x^2 – 73x – 57$
b $6x^2 – 60x – 146$

7 **a** **i** $x^2 + 2x + 1$ **ii** $x^3 + 3x^2 + 3x + 1$
iii $x^4 + 4x^3 + 6x^2 + 4x + 1$
b $11^2 = 121$, $11^3 = 1331$, $11^4 = 14\,641$
c The digits are the same as the coefficients

8 **a** $x^3 + 9x^2 + 27x + 27$
b 27.027 009 001

9 **a** $2x^3 + 3x^2 – 29x + 30$
b $3x^3 + 11x^2 + 8x – 4$

10 **a** $24x^3 + 26x^2 – 173x + 105$
b $50x^3 – 315x^2 + 228x – 44$
c $27x^3 – 108x^2 + 144x – 64$

11 $82 – 5x – 32x^2$

Exercise 8J

1 **a** $(x + 2)(x + 3)$ **b** $(t + 1)(t + 4)$
c $(m + 2)(m + 5)$ **d** $(k + 4)(k + 6)$
e $(p + 2)(p + 12)$ **f** $(r + 3)(r + 6)$
g $(w + 2)(w + 9)$ **h** $(x + 3)(x + 4)$
i $(a + 2)(a + 6)$ **j** $(k + 3)(k + 7)$
k $(f + 1)(f + 21)$ **l** $(b + 8)(b + 12)$
m $(t – 2)(t – 3)$ **n** $(d – 4)(d – 1)$
o $(g – 2)(g – 5)$ **p** $(x – 3)(x – 12)$
q $(c – 2)(c – 16)$ **r** $(t – 4)(t – 9)$
s $(y – 4)(y – 12)$ **t** $(j – 6)(j – 8)$

2 **a** $(p-3)(p-5)$ **b** $(y+6)(y-1)$
c $(t+4)(t-2)$ **d** $(x+5)(x-2)$
e $(m+2)(m-6)$ **f** $(r+1)(r-7)$
g $(n+3)(n-6)$ **h** $(m+4)(m-11)$
i $(w+4)(w-6)$ **j** $(t+9)(t-10)$
k $(h+8)(h-9)$ **l** $(t+7)(t-9)$
m $(d+1)^2$ **n** $(y+10)^2$
o $(t-4)^2$ **p** $(m-9)^2$
q $(x-12)^2$ **r** $(d+3)(d-4)$
s $(t+4)(t-5)$ **t** $(q+7)(q-8)$

3 $(x+2)(x+3)$, giving areas of $2x$ and $3x$, or $(x+1)(x+6)$, giving areas of x and $6x$.

4 **a** $x^2+(a+b)x+ab$
b **i** $p+q=7$ **ii** $pq=12$
c 7 can only be 1 × 7 and 1 + 7 ≠ 12

5 **a** 440
b **i** $(x+3)(x+1)$ **ii** 22 × 20 = 440

6 **a** $(x^2-3)(x^2-8)$ **b** $(y^5-104)(y^5+4)$
c $(z^{1728}-864)(z^{1728}+2)$

Exercise 8K

1 **a** $(x+3)(x-3)$ **b** $(t+5)(t-5)$
c $(m+4)(m-4)$ **d** $(3+x)(3-x)$
e $(7+t)(7-t)$ **f** $(k+10)(k-10)$
g $(2+y)(2-y)$ **h** $(x+8)(x-8)$
i $(t+9)(t-9)$

2 **a** x^2
b **i** $(x-2)$ **ii** $(x+2)$
iii x^2 **iv** 4
c A + B – C = x^2-4, which is the area of D, which is $(x+2)(x-2)$.

3 **a** $x^2+4x+4-(x^2+2x+1)=2x+3$
b $(a+b)(a-b)$
c $(x+2+x+1)(x+2-x-1)=(2x+3)(1)=2x+3$
d The answers are the same.
e $4x$

4 **a** $(x+y)(x-y)$ **b** $(x+2y)(x-2y)$
c $(x+3y)(x-3y)$ **d** $(3x+1)(3x-1)$
e $(4x+3)(4x-3)$ **f** $(5x+8)(5x-8)$
g $(2x+3y)(2x-3y)$ **h** $(3t+2w)(3t-2w)$
i $(4y+5x)(4y-5x)$

5 **a** $(11x^3-3y^3)(11x^3+3y^3)$
b $(5m^5-9n^9)(5m^5+9n^9)$
c $(24p^{288}-31q^{144})(24p^{288}+31q^{144})$

6 **a** $(3x-1)(3x+1)$ **b** 29 and 31

7 **a** $(2x-7)(2x+7)$ **b** 3, 23 and 193

Exercise 8L

1 **a** $(2x+1)(x+2)$ **b** $(7x+1)(x+1)$
c $(4x+7)(x-1)$ **d** $(3t+2)(8t+1)$
e $(3t+1)(5t-1)$ **f** $(4x-1)^2$
g $3(y+7)(2y-3)$ **h** $4(y+6)(y-4)$
i $(2x+3)(4x-1)$ **j** $(2t+1)(3t+5)$
k $(x-6)(3x+2)$ **l** $(x-5)(7x-2)$

2 $4x+1$ and $3x+2$

3 **a** All the terms in the quadratic have a common factor of 6.
b $6(x+2)(x+3)$. This has the highest common factor taken out.

4 $(3x-1)(x+16)$; 1230

5 **a** $(33x+1)(x-2)$ **b** 100 × 1 = 100

6 $(3x-20)$

7 $12x^2+14x-38$; $10x+2$

Exercise 8M

1 $k=\frac{T}{3}$

2 $y=X+1$

3 $p=3Q$

4 $r=\frac{A-9}{4}$

5 **a** $m=p-t$ **b** $t=p-m$

6 $m=gv$

7 $m=\sqrt{t}$

8 $l=\frac{P-2w}{2}$

9 $p=\sqrt{m-2}$

10 **a** –40 – 32 = –72, –72 ÷ 9 = –8, 5 × –8 = –40
b 68 – 32 = 36, 36 ÷ 9 = 4, 4 × 5 = 20
c $F=\frac{9}{5}C+32$

11 Average speeds: outward journey = 72 kph, return journey = 63 kph, taking 2 hours. He was held up for 15 minutes.

12 $r=C/2\pi$, $A=\pi r^2=\pi C^2/4\pi^2=C^2/4\pi$

13 **a** $y=\frac{5x-75}{9}$ **b** Pupil's own checks
c $y=\frac{7x-40}{10}$ **d** Marlon is incorrect

14 **a** $a=\frac{v-u}{t}$ **b** $t=\frac{v-u}{a}$

15 $d=\sqrt{\frac{4A}{\pi}}$

16 **a** $y=\frac{x+w}{5}$ **b** $w=5y-x$

17 $p=\sqrt{\frac{k}{2}}$

18 **a** $t=u^2-v$ **b** $u=\sqrt{v+t}$

19 **a** $w=K-5n^2$ **b** $n=\sqrt{\frac{K-w}{5}}$

20 a $D = \frac{P + Y(K - U)}{3(K - U)}$ or $\frac{\frac{P}{K-U} + Y}{3}$

b 16

Review questions

1 a $20x + 16$ **b** $5x + 4$

2 a $c = \frac{R - 3d - 5}{7}$ **b** 6

3 a i 3.5 ml **ii** 3.7 ml **iii** 3.84 ml
b i 22 **ii** 38 **iii** 90

4 13.5 m^2

5 a $2\pi r(r + h)$ **b** $h = \frac{A - 2\pi r^2}{2\pi r}$
c 5 cm

6 a $x = \frac{A - y}{0.01y}$ **b** $y(1 + 0.01x)$
c 38.36 g

7 $x = 5$

8 20 m

9 a $\frac{19-4x}{5x+18}$ **b** $\frac{18-4x}{5x+17}$

10 a $2(x - 8)$ **b** $x(x - 16)$
c $(x - 4)(x + 4)$ **d** $(x - 7)(x - 9)$

11 a 3 × 15 × 4 = 180 **b** $6x^2 - 51x + 90$

12 a i $15x^2 - 19x - 56$ **ii** $16x - 2$
b 162.25 cm^2

13 a $4x^2 + 4x + 1$ **b** 441 **c** 437

14 a $a^3 + 3a^2b + 3ab^2 + b^3$
b $8x^3 + 36x^2 + 54x + 27$ **c** 27.543 608

15 a i $12x + 48$ **ii** $6x^2 + 48x + 94$
iii $x^3 + 12x^2 + 47x + 60$
b surface area = 1348 cm^2 and volume = 3360 cm^3

16 a $12x^2 - xy - 35y^2$ **b** $(3x + 7y)(2x - 5y)$

17 ±10, ±11, ±14, ±25

18 a $(2x + 3)(x + 2)$
b i 276 **ii** 20 706 **iii** 6.0702

Chapter 9 – Geometry and measures: Length, area and volume

Exercise 9A

1 a 8 cm, 25.1 cm, 50.3 cm^2
b 5.2 m, 16.3 m, 21.2 m^2
c 6 cm, 37.7 cm, 113 cm^2
d 1.6 m, 10.1 m, 8.04 m^2

2 a 5π cm **b** 8π cm
c 18π m **d** 12π cm

3 a 25π cm^2 **b** 36π cm^2
c 100π cm^2 **d** 0.25π m^2

4 8.80 m

5 4 complete revolutions

6 1p : 3.1 cm^2, 2p : 5.3 cm^2, 5p : 2.3 cm^2, 10p : 4.5 cm^2

7 0.83 m

8 38.6 cm

9 Claim is correct (ratio of the areas is just over 1.5 : 1)

10 a 18π cm^2 **b** 4π cm^2

11 9π cm^2

12 Divide 31.3 by to get about 10 m. This is the diameter of the tree. Is your classroom smaller than 10 m × 10 m? It probably isn't, but you need to check.

13 45 complete revolutions

14 a 2π
b I 8π **ii** 18π **iii** 32π

c $A = 2 \times r^2 \times \pi$

Exercise 9B

1 a 96 cm^2 **b** 70 cm^2 **c** 20 m^2
d 125 cm^2 **e** 10 cm^2 **f** 112 m^2

2 No, she has used the sloping side instead of the perpendicular height. It should be 6 × 4 = 24 cm^2

3 Each parallelogram has an area of 30 cm^2. The height of each is 5 cm so the length of each must be 6 cm. x = 6 + 4 + 6 = 16 cm so Freya is incorrect.

4 **a** 500 cm^2 **b** 3 × 5 = 15

Exercise 9C

1 a 30 cm^2 **b** 77 cm^2 **c** 24 cm^2
d 42 cm^2 **e** 40 m^2 **f** 6 cm
g 3 cm **h** 10 cm

2 Area = 15 cm^2

3 a 36.25 cm^2
b 61.2 cm^2
c 90 m^2

4 The area of the parallelogram is $\frac{(a+b)}{h}$. This is the same as two trapezia.

5 Two of 20 cm^2 and two of 16 cm^2

6 a 57 m^2 **b** 702.5 cm^2 **c** 84 m^2

7 trapezium area = 56, square area = 9, shaded area = 56 – 9 = 47 m^2

8 4, because the total area doubled is about 32 m^2

9 80.2%

10 1 100 000 km^2

11 160 cm^2

12 **a** many possible correct answers, e.g. base 6 cm, top 4 cm, height 5 cm. Shaded area is 8 π trapezium must be the same

b the dimensions cannot be exact due to the value of π in the area of the circle

Exercise 9D

1 **a i** 5.59 cm **ii** 22.3 cm^2
b i 8.29 cm **ii** 20.7 cm^2
c i 16.3 cm **ii** 98.0 cm^2
d i 15.9 cm **ii** 55.6 cm^2

2 2π cm, 6π cm^2

3 **a** 73.8 cm **b** 20.3 cm

4 area of sector = $\frac{1}{4} \times \pi \times 8^2 = 16\pi$,

area of circle = $\pi \times 4^2 = 16\pi$

5 **a** 107 cm^2
b 173 cm^2

6 43.6 cm

7 **a** $\frac{180}{\pi}$

b If arc length is 10 cm, distance along chord joining the two points of the sector on the circumference will be less than 10 cm, so angle at centre will be less than 60°

8 **a** 66.8° **b** 10 cm^2

9 Let sector have radius R and arc length C, the angle of the sector is found by

$\theta = \frac{360 \times C}{2 \times \pi \times R}$

and so the area will be $\frac{360 \times C \times \pi \times R^2}{2 \times \pi \times R \times 360}$

$= \frac{CR}{2}$

10 $(36\pi - 72)$ cm^2

11 36.5 cm^2

12 16 cm (15.7)

13 Each square has side length of r

Shaded part of square $X = r^2 - \frac{1}{4}r^2$

$= r^2(1 - \frac{1}{4})$

In square Y, the four quarter circles will join together to give an area of radius $\frac{1}{2}$r, so shaded area

in Y = $r^2 - \left(\frac{r}{2}\right)\pi^2 = r^2 - \frac{1}{4}r^2 = r^2(1 - \frac{1}{4}\pi)$,

which is the same as square X.

Exercise 9E

1 **a i** 21 cm^2 **ii** 63 cm^3
b i 48 cm^2 **ii** 432 cm^3
c i 36 m^2 **ii** 324 m^3

2 **a** 432 m^3 **b** 225 m^3 **c** 1332 m^3

3 **a** A cross-section parallel to the side of the pool always has the same shape.

b About $3\frac{1}{2}$ hours

4 $V = \frac{1}{2}(1.5 + 3) \times 1.7 \times 2 = 7.65$ m^3

5 $27 = 3 \times 3 \times 3$, $27 + 37 = 64 = 4^3$, $4 - 3 = 1$. Hence the side length is 1 small cube longer, hence 2 cm longer

6 **a i** 21 cm^3 **ii** 210 cm^3
b i 54 cm^2 **ii** 270 cm^2

7 146 cm^3

8 78 m^3 (78.3 m^3)

9 327 litres

10 10.2 tonnes

11 She was silly because 188160 is simply all the numbers multiplied together. The volume is 672 cm^3

Exercise 9F

1 **a i** 226 cm^3 **ii** 207 cm^2
b i 14.9 cm^3 **ii** 61.3 cm^2
c i 346 cm^3 **ii** 275 cm^2
d i 1060 cm^3 **ii** 636 cm^2

2 **a i** 72π cm^3 **ii** 48π cm^2
b i 112π cm^3 **ii** 56π cm^2
c i 180π cm^3 **ii** 60π cm^2
d i 600π m^3 **ii** 120π m^2

3 Volume = $\pi \times (0.3)^2 \times 4.2 = 0.378\pi$

Cost = $0.378 \times \pi \times$ £67.50 = £80.16 which is £80 to 2sf

4 1.23 tonnes

5 Label should be less than 10.5 cm wide so that it fits the can and does not overlap the rim and more than 23.3 cm long to allow an overlap.

6 Volume = $\pi \times 32.5^2 \times 100 = 331830.7$ cm^3

1 litre = 1000 cm^3

volume = 331830.7 ÷ 1000 = 331.8307 litres = 332 litres (3 sf)

7 There is no right answer. Students could start with the dimensions of a real can. Often drinks cans are not exactly cylindrical. One possible answer is height of 6.6 cm and diameter of 8 cm.

8 7.78 g/cm^3

9 About 127 cm

10 A diameter of 10 cm and a length of 5 cm give a volume close to 400 cm^3 (0.4 litres).

Exercise 9G

1 **a** 56 cm^3 **b** 168 cm^3 **c** 1040 cm^3
d 84 cm^3 **e** 160 cm^3

2 $\frac{1}{3}$ base area $\times h = \frac{1}{3} \times 9 \times 9 \times 10 = 270$ cm^3

3 **a** Put the apexes of the pyramids together. The 6 square bases will then form a cube.
b If the side of the base is *a* then the height will be $\frac{1}{2}a$.
Total volume of the 6 pyramids is a^3.
Volume of one pyramid is $\frac{1}{6}a^3$ =
$\frac{1}{3} \times \frac{1}{2} \times a \times a^2 = \frac{1}{3} \times$ height × base area

4 6.9 m ($\frac{1}{3}$ height of pyramid)

5 **a** 73.3 m^3 **b** 45 m^3 **c** 3250 cm^3

6 208 g

7 1.5 g

8 $\frac{1}{3} \times 6.4 \times 6.4 \times H = 81.3$

So $H = \frac{3 \times 81.3}{6.4 \times 6.4} = 5.954 = 6.0$ (2sf)

9 14.4 cm

10 Volume of pyramid = $\frac{1}{3} \times 6 \times 9 \times 15 = 270$ cm^3

Volume of part cut off top = $\frac{1}{3} \times 3 \times 2 \times 5$
= 10 cm^3
So frustum = 270 – 10 = 260 cm^3
Hence $\frac{\text{volume of frustrum}}{\text{volume of pyramid}} = \frac{260}{270} = \frac{26}{27}$
therefore, Hannah is correct.

Exercise 9H

1 **a** **i** 3560 cm^3 **ii** 1430 cm^2
b **i** 314 cm^3 **ii** 283 cm^2
c **i** 1020 cm^3 **ii** 679 cm^2

2 935 g

3 Total area = $\pi rl + \pi r^2 = \pi \times 3 \times l + \pi \times 3^2$
= $(3l + 9)\pi = 24\pi$
So $3l + 9 = 24$, so $3l = 24 - 9 = 15$
$l = \frac{15}{3} = 5$

4 **a** 816π cm^3 **b** 720π mm^3

5 24π cm^2

6 **a** 4 cm **b** 6 cm
c Various answers, e.g. 60° gives 2 cm, 240° gives 8 cm

7 If radius of base is *r*, slant height is 2*r*.
Area of curved surface = $r \times 2r\pi = 2r^2\pi$, area of base = πr^2

8 2.7 g/cm^3

9 2.81 cm

10 252π cm^2

Exercise 9I

1 **a** 36π cm^3 **b** 288π cm^3 **c** 1330π cm^3 (3 sf)

2 **a** 36π cm^2 **b** 100π cm^2 **c** 196π cm^2

3 65 400 cm^3, 7850 cm^2

4 **a** 1960 cm^2 (to 3sf) **b** 7444 cm^3 (to nearest unit)

5 125 cm

6 6232

7 7.8 cm

8 **a** The surface area, because this is the amount of material (leather or plastic) needed to make the ball
b Surface area can vary from about 1470 cm^2 to 1560 cm^2, difference of about 90 cm^2. This seems surprisingly large.

9 48%

10 Radius of sphere = base radius of cylinder = *r*, height of cylinder = 2*r* Curved surface area of cylinder = circumference × height = $2\pi r \times 2r = 4\pi r^2$ = surface area of sphere

Review questions

1 29.4 cm^2

2 721 cm^2

3 5740 cm^3(to 3sf)

4 610 g (2sf)

5 17.5 cm

6 360 g

7 56.5 cm

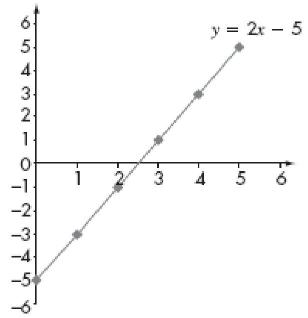
8 Call length of square 2*x*, so that radius of arcs is *x*.
Then area of square = $4x^2$
Area of each semicircle = $\frac{1}{2}\pi x^2$ so area of 4 semi circles is $2x^2$
Area of shaded part is: area of 4 semicircles – area of square = $(2\pi - 4)x^2$
So percentage shaded = $\frac{2\pi - 4}{4x^2} x^2 \times 100$ =
$\frac{2\pi - 4}{4} \times 100 = 57\%$

9 $\frac{1}{3}$

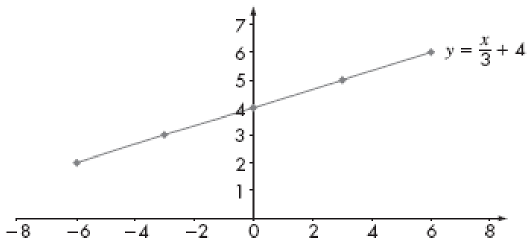
Chapter 10 – Algebra: Linear graphs

Exercise 10A

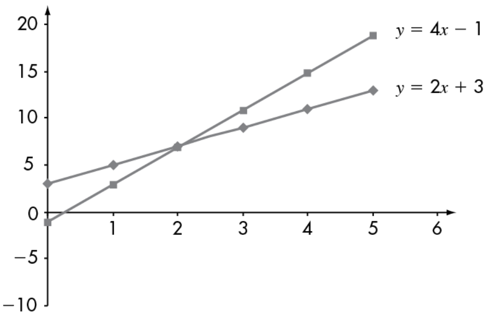
1



2

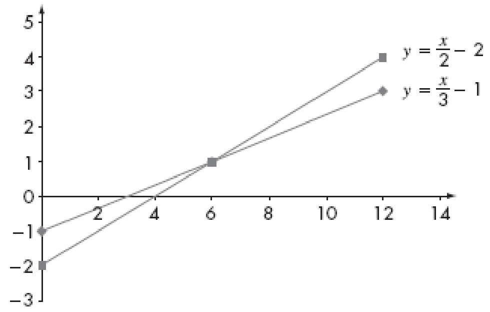


3 a

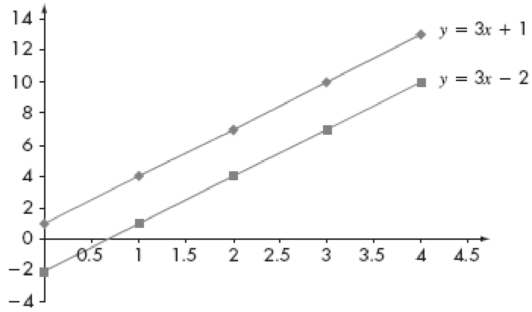


b (2, 7)

4 a



b



4 c lines in part **a** intersect at (6, 1), lines in part **b** don't intersect because they are parallel

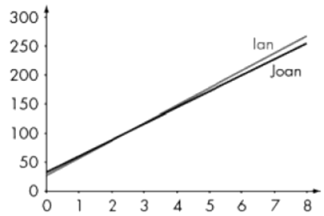
5 a Line isn't straight

b

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7

Correct line drawn

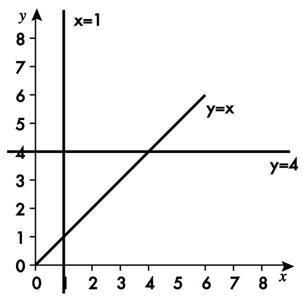
6 a



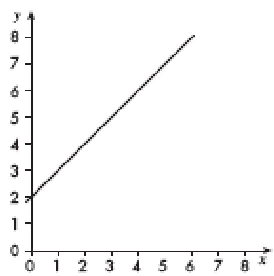
b Ian, Ian only charges £85, whilst Joan charges £90 for a 2-hour job.

7 a Jada's method

8 a



b 4.5 units squared



9

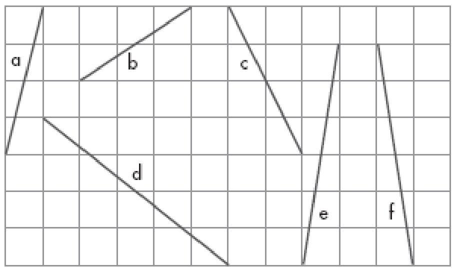
10 18 units squared

Exercise 10B

1 **a** 2 **b** $\frac{1}{3}$ **c** −3 **d** 1 **e** −2

f $-\frac{1}{3}$ **g** 5 **h** −5 **i** $\frac{1}{5}$ **j** $-\frac{3}{4}$

2



3 **a** Both answers are correct
b Generally the bigger the triangle the more accurate the answer, so Brianna

4 **a** ladder might slip
b ladder might topple
c A, B, E and F satisfy the safety regulations; C and D do not

5 **a** 0.5 **b** 0.4 **c** 0.2 **d** 0.1 **e** 0

6 **a** $1\frac{2}{3}$ **b** 2 **c** $3\frac{1}{3}$ **d** 10 **e** ∞

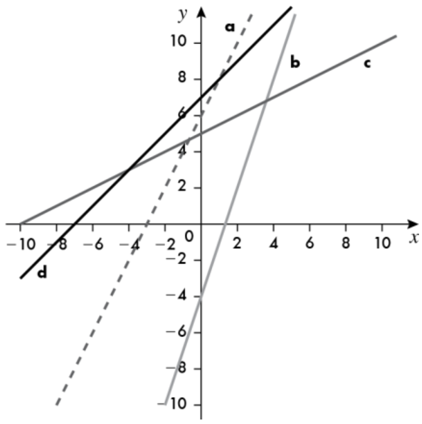
7 Raisa has misread the scales. The second line has four times the gradient (2.4) of the first (0.6)

8 **a** $\frac{3}{8}$
b $\frac{2}{5}$
c Although the puzzle appears to be a right-angled triangle, because the gradients of the smaller triangles are different there is actually a bend in the large hypotenuse, so it is actually a quadrilateral. In the first diagram it has a concave angle and in the second diagram the equivalent angle is convex, and the area of the square hole is spread out between them.

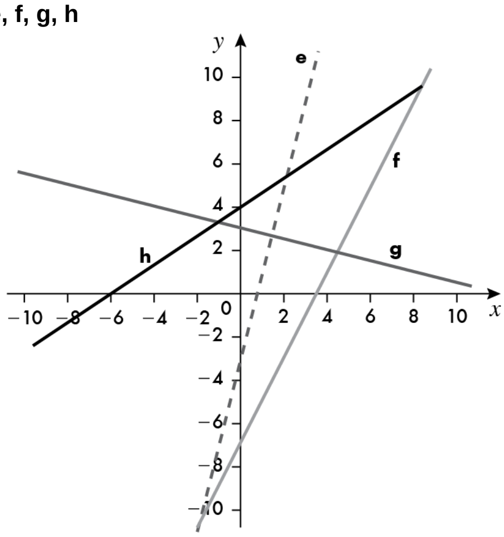
9 0, 2, –1, $\frac{1}{2}$, $-\frac{3}{2}$

Exercise 10C

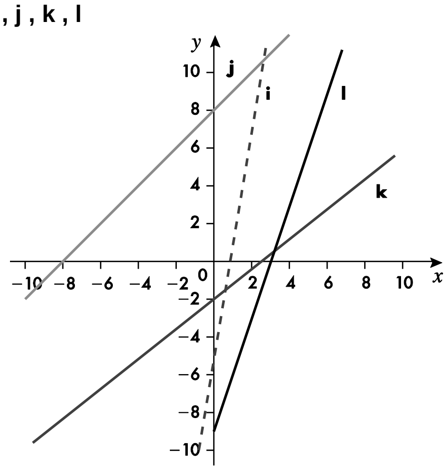
1 **a, b, c, d**



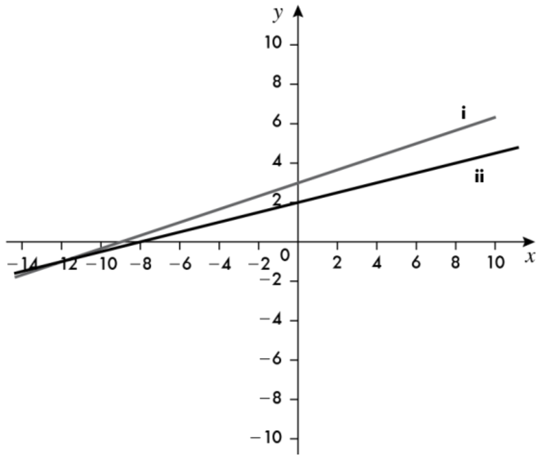
e, f, g, h



i, j , k , l



2 a



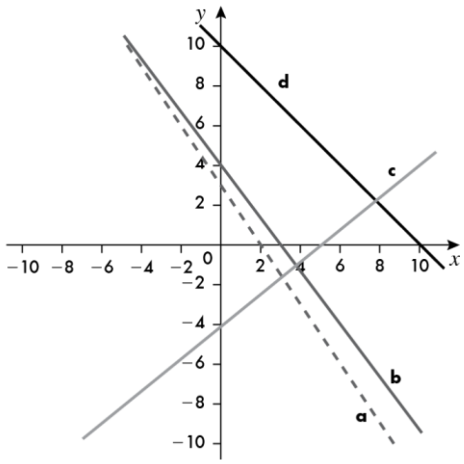
b (−12, −1)

3 a They have the same gradient (3).
b They intercept the y-axis at the same point (0, −2).
c (−1, −4)

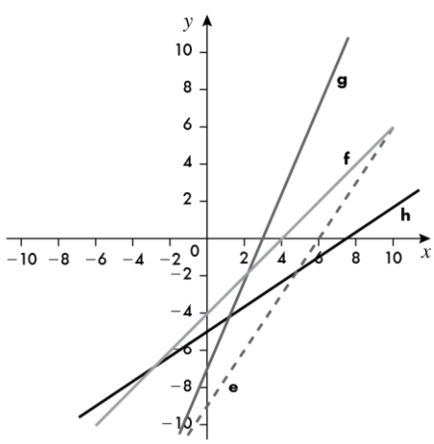
4 a −2 **b** $\frac{1}{2}$ **c** 90°
d Negative reciprocal **e** $-\frac{1}{3}$

Exercise 10D

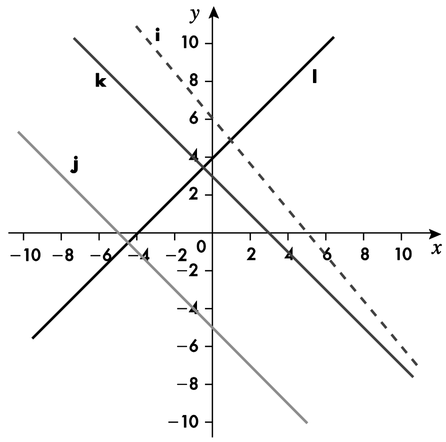
1 a, b, c, d



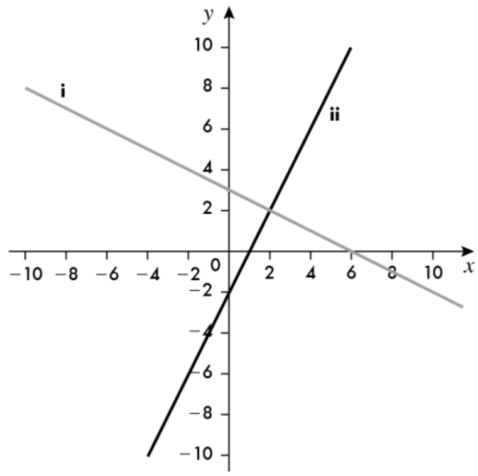
e, f, g, h



i, j, k, l



2 a



b (2, 2)

3 a Intersect at (6, 0)
b Intersect at (0, −3)
c Parallel
d $-2x + 9y = 18$

4 a vi **b** iii **c** v **d** ii **e** i **f** iv

5 $y = -6x$; $y = 3x + 4$; $2y - 5x = 10$; $2y - x = 7$; $y = 4$

6 $3x + 2y = 18$ and $y = 9 - x$

7 a i $x = 3$ **ii** $x - y = 4$ **iii** $y = -3$
iv $x + y = -4$ **v** $x = -3$ **vi** $y = x + 4$
b i −3 **ii** $\frac{1}{3}$ **iii** $-\frac{1}{3}$

8 Cover-up method for $2x + y = 10$ and gradient-intercept method for $y = 11 - 2x$

Exercise 10E

1 a $y = \frac{7}{5}x - 2$ or $5y = 7x - 10$
b $y = 2x$ **c** $2y = x + 6$

2 a i $y = 2x + 1$, $y = -2x + 1$
ii Reflection in y-axis (and $y = 1$)
iii Different sign
b i $5y = 2x - 5$, $5y = -2x - 5$
ii Reflection in y-axis (and $y = -1$)
iii Different sign

c **i** $y = x + 1$, $y = -x + 1$
ii Reflection in y-axis (and $y = 1$)
iii Different sign

3 **a** x-coordinates go from $2 \to 1 \to 0$ and y-coordinates go from $5 \to 3 \to 1$.
b x-step between the points is 1 and y-step is **2.**
c $y = 3x + 2$

4 **a** $y = -x + 1$
b $5y = -2x - 5$
c $y = -\frac{3}{2}x - 3$ or $2y = -3x - 6$

5 **a** **i** $2y = -x + 1$, $y = -2x + 1$
ii Reflection in $x = y$
iii Reciprocal of each other
b **i** $2y = 5x + 5$, $5y = 2x - 5$
ii Reflection in $x = y$
iii Reciprocal of each other
c **i** $y = 2$, $x = 2$
ii Reflection in x = y
iii Reciprocal of each other (reciprocal of zero is infinity)

6 All of the lines except $y = \frac{1}{4}x + 9$

7 **a** $y = -3x + 5$ **b** $y = 2x - 4$
c $y = 8x - 3$ **d** $y = 25 - 2x$
e $y = \frac{2}{3}x - 1$

8 $5x + 6y = 30$

9 Chris is correct. The equation of the line is $y = \frac{1}{2}x + 2$ and (12, 8) satisfies the equation

10 **a** **i** $x + y = 100$ **ii** $k = 1$
b **i** $x = 46$ **ii** $k = 46$
c **i** $y = 2x + 1$ **ii** $k = 60$
d **i** $y = x + 19$ **ii** $k = -17$

11 (4, 11)

Exercise 10F

1 **a** Anya: CabCo £8.50, YellaCabs £8.40, so YellaCabs is best; Bettina: CabCo £11.50, YellaCabs £11.60, so CabCo is best; Calista: CabCo £10, YellaCabs £10, so either
b If they shared a cab, the shortest distance is 16 km, which would cost £14.50 with CabCo and £14.80 with Yellacabs.

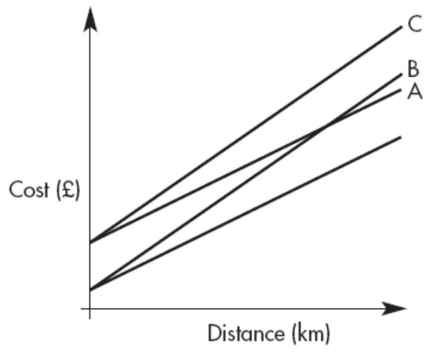
2 **a** **i** $8\frac{1}{4}$ kg **ii** $2\frac{1}{4}$ kg
iii 9 lb **iv** 22 lb
b 2.2 lb
c Read off the value for 12 lb (5.4 kg) and multiply this by 4 (21.6 kg)

3 **a** 32° F
b $\frac{9}{5}$ (Take gradient at C = 10° and 30°)
c $F = \frac{9}{5}C + 32$

4 **a** 0.07 (Take gradient at U = 0 and 500)
b £10
c C = £(10 + 0.07U) or Charge = £10 + 7p/unit

5 **a** \$1900 – \$1260 = \$640
b **i** \$7500 **ii** £3680

6



7 $y = 2x + 15$ $0 < x \le 5$
$y = x + 20$ $5 < x \le 12$
$y = \frac{1}{2}x + 26$ $12 < x \le 22$

Exercise 10G

1 (4, 1)

2 (2, 3)

3 (3, 10)

4 (–2, 6)

5 (–6, –9)

6 (1, –1)

7 (2, 6)

8 (2, 8)

9 $(7\frac{1}{2}, 3\frac{1}{2})$

10 $x + 2y = 9.5$, $2x + y = 8.5$
Graphs intersect at (2.5, 3.5), so a cheesecake costs £2.50 and a gâteau costs £3.50.

11 **a** P and R **b** R and S
c P and Q **d** Q and S

12 (0, 0), (–3, 3), (–3, –3), (–3, 2), (–2, 2), (2, 2)

13 **a** No solutions, lines are parallel
b Infinite solutions, lines are same
c One solution, lines intersect once

Exercise 10H

1 **a** Line A does not pass through (0, 1).
b Line C is perpendicular to the other two.
c (i)

2 **a** $-\frac{1}{2}$ **b** $\frac{1}{3}$ **c** -2 **d** $\frac{3}{2}$ **e** $-\frac{2}{3}$ **f** $-\frac{3}{4}$

3 $y = 3x + 5$, $x + 3y = 10$, $y = 8 - \frac{1}{3}x$, $y = 3(x + 2)$

4 $x = 6$ and $y = -2$
$x + y = 5$ and $y = x + 4$
$y = 8x - 9$ and $y = -\frac{1}{8}x + 6$
$2y = x + 4$ and $2x + y = 9$
$5y = 2x + 15$ and $2y + 5x = 2$
$y = 0.1x + 2$ and $y = 33 - 10x$

5 **a** $y = \frac{1}{2}x - 2$ **b** $y = -x + 3$
c $y = -\frac{1}{3}x - 1$ **d** $y = 3x + 5$

6 **a** –4
b $\frac{1}{4}$
c (11, 7)
d $y = \frac{1}{4}x + c$
Substitute in (11, 7) and solve to get c = $\frac{17}{4}$, so $4y - x = 17$

7 $y = -\frac{1}{4}x + 2$

8 **i** **a** AB: $-\frac{1}{5}$, BC: 1, CD: $-\frac{1}{5}$, DA: 1
b Parallelogram (two pairs of parallel sides)
ii **a** AB: $\frac{2}{3}$, BC: $-\frac{3}{2}$, CD: $\frac{2}{3}$, DA: $-\frac{3}{2}$
b Rectangle (two pairs of perpendicular sides)
iii **a** AB: $\frac{2}{5}$, BC: $\frac{1}{4}$, CD: $\frac{2}{5}$, DA: 1
b Trapezium (one pair of parallel sides)

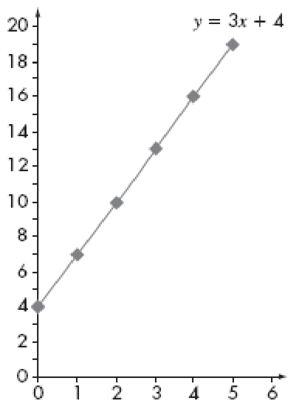
9 $y = -\frac{1}{2}x + 5$

10 **a** $y = 3x - 6$
b Bisector of AB is $y = -2x + 9$, bisector of AC is $y = \frac{1}{2}x + \frac{3}{2}$, solving these equations shows the lines intersect at (3, 3).
c (3, 3) lies on $y = 3x - 6$ because $(3 \times 3) - 6 = 3$

11 (3, 10)

Review questions

1



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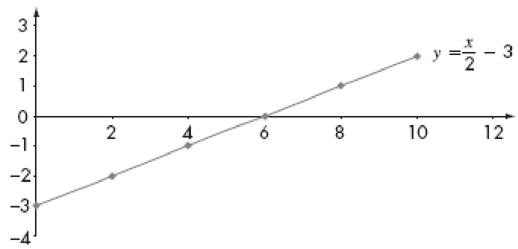

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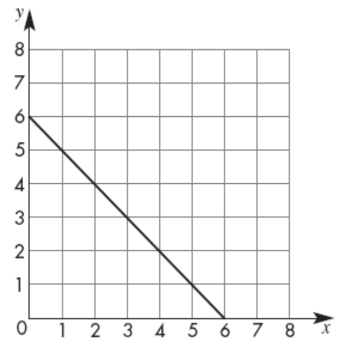


4 **a** $\frac{1}{2}$ (Take gradient at N = 0 and 500)
b £50
c C = £(50 + $\frac{N}{2}$) or £50 + 50p/person

5 **a** $\frac{1}{10}$
b 24.5 cm
c 0.1 cm or 1 mm
d $L = 24.5 + \frac{W}{10}$ or Length = 24.5 + 1 mm/kg

6 **a** (5, 5) **b** (1, 5) **c** (3, 16)

7 **a** 2 **b** $y = 2x + 2$ **c** $y = -\frac{1}{2}x + 7$

8 **a** Reflection in y-axis ($x = 0$); reflection in $y = 1$ (rotations also possible)
b Rotation 90° clockwise; rotation 90° anticlockwise about the point (0, 1) (reflections also possible)

9 30 square units

10 36 square units

11 (7, 1)

Chapter 11 – Geometry: Right-angled triangles

Exercise 11A

1 Students' own diagrams

2 Possible answers include multiples of 3, 4, 5; multiples of 5, 12, 13; multiples of 7, 24, 25; multiples of 8, 15, 17

3 **a** 10.3 cm **b** 5.9 cm **c** 8.5 cm
d 20.6 cm **e** 18.6 cm **f** 17.5 cm
g 13 cm **h** 5 cm

4 **a** √8, √12, √16
b Add 4 to 16 to give H_4 as √20

5 The square in the first diagram and the sum of the two squares in the second have the same area.

Exercise 11B

1 **a** 15 cm **b** 14.7 cm
c 6.3 cm **d** 18.3 cm

2 **a** 20.8 m **b** 15.5 cm
c 15.5 m **d** 12.4 cm

3 **a** 5 m **b** 6 m
c 3 m **d** 50 cm

4 There are infinite possibilities, e.g. any multiple of 3, 4, 5 such as 6, 8, 10; 9, 12, 15; 12, 16, 20; multiples of 5, 12, 13; multiples of 7, 24, 25 and of 8, 15, 17.

5 498.4 cm^2

6 Any of (0, 0) , (5, 5), (2, 0), (5, 3), (2, 8), (0, 8), (–3, 3), (–3, 5) are the most likely points

7 Use Pythagoras' theorem to find a few possible dimensions of the rectangle, then plot a graph of one side length against the area. You will see that 50 is the highest the area will ever get to.

8 The large square is 17 by 17 giving 289 square units.
The red and yellow triangles all have shorter lengths of 5 and 12, with an area of 30 square units.
The area of the inner square (green and yellow) must be 289 – 4 × 30 = 169, so the hypotenuse of the yellow triangles must be √169 = 13
You can see that $5^2 + 12^2 = 13^2$

Exercise 11C

1 No. The foot of the ladder is about 6.6 m from the wall.

2 2.06 m

3 11.3 m

4 About 17 minutes, assuming it travels at the same speed.

5 127 m – 99.6 m = 27.4 m

6 4.58 m

7 **a** 3.87 m **b** 1.74 m

8 3.16 m

9 This creates a right-angled triangle with two short sides of 5 and 12. Use Pythagoras' theorem to show length of line = $\sqrt{(5^2 + 12^2)} = 13$

10 **a** 4.85 m
b 4.83 m (There is only a small difference.)

11 Yes, because $24^2 + 7^2 = 25^2$

12 6 cm

13 He is partly correct. The perimeter must be larger than 20 cm or the rectangle has no width, and the area is largest when it's a square, giving a perimeter of 28.3 cm (3 sf). So he should have said the perimeter is between 20 and 28.3 cm.

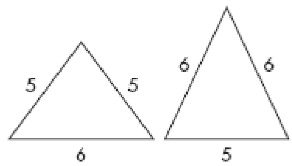
Exercise 11D

1 **a** 32.2 cm^2 **b** 2.83 cm^2 **c** 50.0 cm^2

2 22.2 cm^2

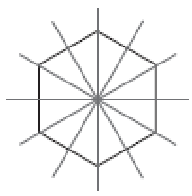
3 15.6 cm^2

4 **a**



b The areas are 12 cm^2 and 13.6 cm^2 respectively, so triangle with 6 cm, 6 cm, 5 cm sides has the greater area.

5 **a** **b** 166.3 cm^2



6 259.8 cm^2

7 **a** No, areas vary from 24.5 cm^2 to 27.7 cm^2
b No, equilateral triangle gives the largest area.
c The closer the isosceles triangle gets to an equilateral triangle the larger its area becomes.

8 Show the right-angled triangle made with hypotenuse 6.5 m and base 7.4 ÷ 2 = 3.7, giving the height of the triangle as 5.344 cm. Use area = ½ × 7.4 × 5.344 to give 19.7733 which rounds to 19.8 m^2 (3 sf)

9 48 cm^2

10 **a** 10 cm **b** 26 cm **c** 9.6 cm

11 6 or 8 cm

12 Andrew didn't round off any answers until the last calculation, and Olly used a rounded off value to find an intermediate result.

Exercise 11E

1 **a** **i** 14.4 cm **ii** 13 cm **iii** 9.4 cm
b 15.2 cm

2 No, 6.6 m is longest length

3 **a** 20.6 cm **b** 15.0 cm

4 **a** 8.49 m **b** 9 m

5 $10^2 + 10^2 + 10^2 = 300$, $\sqrt{300} = 17.3$ cm (3sf)

6 20.6 cm

7 **a** 11.3 cm **b** 7 cm **c** 8.06 cm

8 AM = $\sqrt{(22.5^2 + 15^2 + 40^2)} = 48.283 = 48.3$ cm (3 sf)

9 21.3 cm

Exercise 11F

1 **a** 0.682 **b** 0.829 **c** 0.922
d 1 **e** 0.707 **f** 0.342
g 0.375 **h** 0

2 **a** 0.731 **b** 0.559 **c** 0.388
d 0 **e** 0.707 **f** 0.940
g 0.927 **h** 1

3 45°

4 **a** **i** 0.574 **ii** 0.574
b **i** 0.208 **ii** 0.208
c **i** 0.391 **ii** 0.391
d Same
e **i** sin 15° is the same as cos 75°
ii cos 82° is the same as sin 8°
iii sin x is the same as cos (90° – x)

5 **a** 0.933 **b** 1.48 **c** 2.38
d Infinite (calculator will give a maths error)
e 1 **f** 0.364 **g** 0.404
h 0

6 **a** 0.956 **b** 0.899 **c** 2.16
d 0.999 **e** 0.819 **f** 0.577
g 0.469 **h** 0.996

7 Has values > 1

8 **a** 4.53 **b** 4.46 **c** 6 **d** 0

9 **a** 10.7 **b** 5.40
c 68.58 **d** 0

10 **a** 3.56 **b** 8.96 **c** 28.4 **d** 8.91

11 **a** 5.61 **b** 11.3 **c** 6 **d** 10

12 **a** 1.46 **b** 7.77 **c** 0.087 **d** 9.33

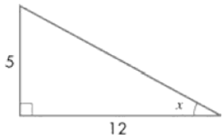
13 **a** 7.73 **b** 48.6 **c** 2.28 **d** 15.2

14 **a** 29.9 **b** 44.8 **c** 20.3 **d** 2.38

15 **a** $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$

b $\frac{7}{25}$, $\frac{24}{25}$, $\frac{7}{24}$

16 You should have drawn a right angled triangle as here. H = 13 since this is a 5, 12, 13 Pythagorean triple. See that opposite = 5 and adjacent = 12.
Hence sin $x = \frac{O}{H} = \frac{5}{13}$ and cos $x = \frac{12}{13}$



Exercise 11G

1 **a** 30° **b** 51.7° **c** 39.8°
d 61.3° **e** 87.4° **f** 45.0°

2 **a** 60° **b** 50.2° **c** 2.6°
d 45.0 **e** 78.5° **f** 45.6°

3 **a** 31.0° **b** 20.8° **c** 41.8°
d 46.4° **e** 69.5° **f** 77.1°

4 **a** 53.1° **b** 41.8° **c** 44.4°
d 56.4° **e** 2.4° **f** 22.6°

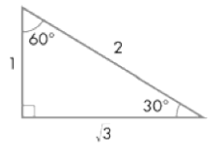
5 **a** 36.9° **b** 48.2° **c** 45.6°
d 33.6° **e** 87.6° **f** 67.4°

6 **a** 31.0° **b** 37.9° **c** 15.9°
d 60.9° **e** 57.5° **f** 50.2°

7 **a** Error message **b** Largest value 1
c Smallest value –1

8 **a** **i** 17.5° **ii** 72.5° **iii** 90°
b Yes

9



Adj = 1, Hyp = 2, hence Opp = $\sqrt{(2^2 - 1^2)} = \sqrt{3}$

a 30 is the other acute angle in the triangle and so for 30, opp = 1 and adj = $\sqrt{3}$, hence tan 30 = $\frac{1}{\sqrt{3}}$

b tan 60 = $\sqrt{3}$

c sin 60 = $\frac{\sqrt{3}}{2}$

d cos 30 = $\frac{\sqrt{3}}{2}$

e sin 30 = $\frac{1}{2}$

10 Adj = 1, Opp = 1, hence Hyp = $\sqrt{(1^2 + 1^2)} = \sqrt{2}$

a $\sin 45 = \frac{1}{\sqrt{2}}$

b $\cos 45 = \frac{1}{\sqrt{2}}$

Exercise 11H

1 **a** 17.5° **b** 22.0° **c** 32.2°

2 **a** 5.29 cm **b** 5.75 cm **c** 13.2 cm

3 **a** 4.57 cm **b** 6.86 cm **c** 100 cm

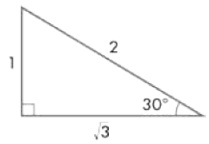
4 **a** 5.12 cm **b** 9.77 cm
c 11.7 cm **d** 15.5 cm

5 **a** 47.2° **b** 5.42 cm
c 13.7 cm **d** 38.0°

6 **a** 6 **b** 15 **c** 30

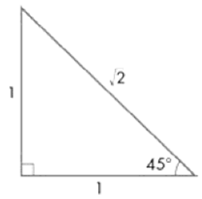
7 **a** $\frac{1}{2}$

b and **c**



8 **a** $\frac{1}{\sqrt{2}}$

b



Exercise 11I

1 **a** 51.3° **b** 75.5° **c** 51.3°

2 **a** 6.47 cm **b** 32.6 cm **c** 137 cm

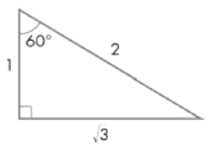
3 **a** 7.32 cm **b** 39.1 cm **c** 135 cm

4 **a** 5.35 cm **b** 14.8 cm
c 12.0 cm **d** 8.62 cm

5 **a** 5.59 cm **b** 46.6°
c 9.91 cm **d** 40.1°

6 **a** 10 **b** 39 **c** 2.5

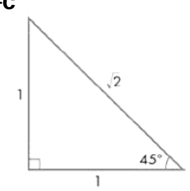
7 **a** $\frac{1}{2}$



b–c

8a $\frac{1}{\sqrt{2}}$

b–c



Exercise 11J

1 **a** 33.7° **b** 36.9° **c** 52.1°

2 **a** 5.09 cm **b** 30.4 cm **c** 1120 cm

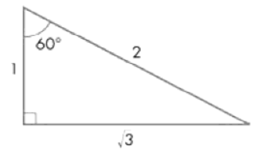
3 **a** 8.24 cm **b** 62.0 cm **c** 72.8 cm

4 **a** 9.02 cm **b** 7.51 cm
c 7.14 cm **d** 8.90 cm

5 **a** 13.7 cm **b** 48.4°
c 7.03 cm **d** 41.2°

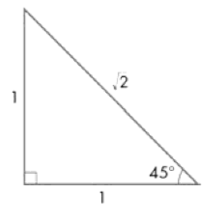
6 **a** 12 **b** 12 **c** 2

7 **a** √3
b



8 **a** 1

b



Exercise 11K

1 **a** 12.6 **b** 59.6 **c** 74.7
d 16.0 **e** 67.9 **f** 20.1

2 **a** 44.4° **b** 39.8° **c** 44.4°
d 49.5° **e** 58.7° **f** 38.7°

3 **a** 67.4° **b** 11.3 **c** 134
d 28.1° **e** 39.7 **f** 263
g 50.2° **h** 51.3° **i** 138
j 22.8

4 **a** Sides of right-hand triangle are sine and cosine
b Pythagoras' theorem
c Students should check the formulae

5

	30°	45°	60°
Sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tangent	$\frac{1}{\sqrt{3}}$	1	√3

Exercise 11L

1 65°

2 The safe limits are between 1.04 m and 2.05 m. The ladder will reach between 5.63 m and 5.90 m up the wall.

3 44°

4 6.82 m

5 31°

6 **a** 25° **b** 2.10 m
c Thickness of wood has been ignored

7 **a** 20° **b** 4.78 m

8 She would calculate 100 tan 23°. The answer is about 42.4 m

9 21.1 m

10 One way is stand opposite a feature, such as a tree, on the opposite bank, move a measured distance, x, along your bank and measure the angle, θ, between your bank and the feature. Width of river is $x \tan \theta$. This of course requires measuring equipment! An alternative is to walk along the bank until the angle is 45° (if that is possible). This angle is easily found by folding a sheet of paper. This way an angle measurer is not required.

Exercise 11M

1 10.1 km

2 22°

3 429 m

4 **a** 156 m
b No. the new angle of depression is $\tan^{-1}\left(\frac{200}{312}\right) = 33°$ and half of 52° is 26°

5 **a** 222 m **b** 42°

6 **a** 21.5 m **b** 17.8 m

7 **a** 13.4 m
b We don't know if the angle of elevation is from Sunil's feet or head. This would make a difference to the answer as we would need to add Sunil's height if the angle was from his head.

8 Cos $\theta = \frac{1}{3}$ so $\cos^{-1} 0.3333 = 70.5°$ (3 sf)

9 The angle is 16° so Cara is not quite correct.

10 William is 137 m away, Isaac is 107 m away.

Exercise 11N

1 **a** 73.4 km **b** 15.6 km

2 **a** 14.7 miles **b** 8.5 miles

3 Draw a diagram representing the relative places. Your diagram will show the angle of the bearing to fly the direct route as $90° + \tan^{-1}(70/120) = 120°$. So the bearing for the direct route is 120°.

4 **a** 59.4 km **b** 8.4 km

5 **a** 15.9 km **b** 24.1 km
c 31.2 km **d** 052°

6 2.28 km

7 235°

8 **a** 66.2 km **b** 11.7 km
c 13.1 km **d** 170°

9 48.4 km on a bearing of 100°

Exercise 11P

1 **a** 5.79 cm **b** 48.2°
c 7.42 cm **d** 81.6 cm

2 9.86 m

3 **a** 36.4 cm^2 **b** 115 cm^2
c 90.6 cm^2 **d** 160 cm^2

4 473 cm^2

5 39.0 cm^2

6 Base radius given by 8 tan 31°, so volume = $\frac{8}{3} \times \pi(8 \tan 31)^2 = 193.57357 = 194$ (3 sf)

Review questions

1 13.6 cm^2

2 2 pm

3 diagonal = $\sqrt{(3^2 + 4^2 + 12^2)} = \sqrt{169} = 13$

4 237°

5 52.3°

6 **a** AX and BY are both radii to the tangents at A and B and so perpendicular to AB, hence parallel. So ABYX is a trapezium.
b Draw in a line, YT, parallel to the base AB, so that T lies on AX. This gives a right-angled triangle with height (7 – 2) cm = 5 cm.
The hypotenuse is (7 + 2) cm = 9 cm, hence the line YT, which is the same length as AB = $\sqrt{(9^2 - 5^2)} = \sqrt{56} = 7.4833…$
The area of the trapezium = AB × (7 + 2)/2 = 7.4833 × 4.5 = 33.6749 = 33.7 cm^2 (3 sf)

7 110 cm^2

8 Using square roots is dependent on remembering that sin 60° = $\frac{\sqrt{3}}{2}$, then calculating this as 0.866 (3 sf).
Using the equilateral triangle will give sin 60° as Height of triangle/10. The height found by Pythagoras as $\sqrt{(10^2 - 5^2)} = \sqrt{75} = 8.660254$, so sin 60° = 0.866 (3 sf).
Both answers are the same.

Chapter 12 – Geometry and measures: Similarity

Exercise 12A

1 **a** Yes, 4
b No, corresponding sides have different ratios.

2 **a** 1 : 3
b Angle R
c BA

3 **a** Angle P **b** PR

4 **a** Same angles
b Angle Q
c AR

5 **a** 8 cm
b 7.5 cm
c x = 6.67 cm, y = 13.5 cm
d x = 24 cm, y = 13 cm
e AB = 10 cm, PQ = 6 cm
f 4.2 cm

6 **a** Sides in same ratio **b** 1 : 3
c 13 cm **d** 39 cm

7 5.2 m

8 Corresponding sides are not in the same ratio, 12 : 15 ≠ 16 : 19.

9 Jay is wrong: DE = 17.5 cm; AC : EC = BA : DE, 5 : 12.5 = 7 : DE, DE = 7 × 12.5 ÷ 5 = 17.5 cm

Exercise 12B

1 **a** ABC and ADE; 9 cm
b ABC and ADE; 12 cm

2 **a** 5 cm
b 5 cm
c x = 60 cm, y = 75 cm
d DC = 10 cm, EB = 8 cm

3 82 m

4 $\frac{pole}{330} = \frac{400}{600}$, pole = 330 × $\frac{4}{6}$ = 220

5 15 m

6 3.3 m

7 1.8 m

8 $\frac{BC}{9} = \frac{9}{6}$, hence BC = 9 × $\frac{9}{6}$ = 13.5 cm

9 c

Exercise 12C

1 **a** 5 cm
b 6 cm
c 10 cm
d x = 6 cm, y = 7.5 cm
e x = 15 cm, y = 21 cm
f x = 3 cm, y = 2.4 cm

2 **a** $\frac{x+12}{12} = \frac{180-1.7}{2-1.7}$, $x = \frac{2136}{0.3}$ = 7120 m, just over 7 km.
b The assumption is that the building, the brick wall and Brad are all standing on the same level.

Exercise 12D

1 **a** **i** 1 : 9 **ii** 4 : 25 **iii** 16 : 49
b **i** 1 : 27 **ii** 8 : 125 **iii** 64 : 343

2

Linear scale factor	Linear ratio	Linear fraction	Area scale factor	Volume scale factor
2	1 : 2	$\frac{2}{1}$	4	8
3	1 : 3	$\frac{3}{1}$	9	27
$\frac{1}{4}$	4 : 1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
5	1 : 5	$\frac{5}{1}$	25	125
$\frac{1}{10}$	10 : 1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

3 135 cm^2

4 **a** 56 cm^2
b 126 cm^2

5 **a** 48 m^2
b 3 m^2

6 **a** 2400 cm^3
b 8100 cm^3

7 Length ratio = 1 : 2, so volume ratio = 1 : 8. So large tin volume = 0.5 × 8 = 4 litres

8 1.38 m^3

9 **a** £6
b Assume that the cost is only based on the volume of paint in the tin.

10 4 cm

11 8 × 60p = £4.80 so it is better value to buy the large tub

12 **a** 3 : 4
b 9 : 16
c 27 : 64

13 720 ÷ 8 = 90 cm^3

Exercise 12E

1 **a** 111 cm^3 **b** 641 cm^3
c 267 cm^3 **d** 426 cm^3

2 **a** Height = 6 cm, Volume = 25 cm^3
b Height = 8 cm, Volume = 51 cm^3
c Height = 4 cm, Mass = 105 g
d Height = 3 cm, Volume = 130 cm^3

3 6.2 cm, 10.1 cm

4 4.26 cm, 6.74 cm

5 $\frac{H^2}{8^2} = \frac{200}{140}$

$H^2 = 64 \times \frac{200}{140} = 91.428571$

$H = \sqrt{91.428571} = 9.56$ (3 sf)

6 3.38 m

7 8.39 cm

8 26.5 cm

9 16.9 cm

10 **a** 4.33 cm, 7.81 cm **b** 143 g, 839 g

11 53.8 kg

12 1.73 kg

13 8.8 cm

14 7.9 cm and 12.6 cm

15 b

Review questions

1 Let height of larger triangle = *h*.

Using similar triangles: $\frac{h}{35-h} = \frac{40}{30}$

Rearrange to $30h = 1400 - 40h$

which gives $70h = 1400$

$h = 20$ cm

So small triangle is 35 – 20 = 15 cm tall.

Thus the difference between the heights is 20 cm – 15 cm = 5 cm

2 **a** For similar shapes, if the ratio of lengths is 1 : x , then the ratio of volumes will be 1 : x^3 , so if ratio of lengths is 1 : 3, the ratio of volumes will be 1 : 3^3 = 1 : 27

b Yes, because if the size (volume) of the plant increases by a factor of 27, the lengths have increased by a factor of 3. Hence the new height should be 4 cm × 3 = 12 cm, which it is.

3 Andrew is correct, Eve has calculated the length of AD (8 cm) so ED should be 2 cm.

4 **a** 6 cm **b** 16 cm^3

5 **a** Area scale factor = $\frac{324}{100}$ = 3.24, length scale factor = $\sqrt{3.24}$ = 1.8, length of cylinder B = 5 × 1.8 = 9

b 933 cm^3

Chapter 13 – Probability: Exploring and applying probability

Exercise 13A

1 **a** $\frac{1}{5}$, $\frac{2}{25}$, $\frac{1}{10}$, $\frac{21}{200}$, $\frac{37}{250}$, $\frac{163}{1000}$, $\frac{329}{2000}$

b 6 **c** 1 **d** $\frac{1}{6}$ **e** 1000

2 **a** $\frac{19}{200}$, $\frac{27}{200}$, $\frac{4}{25}$, $\frac{53}{200}$, $\frac{69}{200}$

b 40

c No, it is weighted towards the side with numbers 4 and 5

3 **a** 32 is too high, unlikely that 20 of the 50 throws between 50 and 100 were 5

b Yes, all frequencies fairly close to 100

4 **a** B **b** B **c** C **d** A **e** B **f** A **g** B **h** B

5 **a** 0.2, 0.25, 0.38, 0.42, 0.385, 0.397

b 80

6 **a** Caryl, most throws **b** 0.39. 0.31, 0.17, 0.14

c Yes, it is more likely to give a 1 or 2

7 Thursday as it had the highest proportion

8 The missing top numbers are 4 and 5; the two bottom numbers are likely to be close to 20.

9 Although you would expect the probability to be close to $\frac{1}{2}$, hence 500 heads, it is more likely that the number of heads is close to 500 rather than actually 500.

10 Roxy is correct, as the expected numbers are: 50, 12.5, 25, 12.5. Sam has not taken into account the fact that there are four red sectors.

Exercise 13B

1 **a** Yes **b** Yes **c** No **d** No **e** Yes **f** Yes

2 Events **a** and **f**

3 $\frac{3}{5}$

4 **a** **i** $\frac{3}{10}$ **ii** $\frac{3}{10}$ **iii** $\frac{3}{10}$ **iv** $\frac{9}{10}$ **v** $\frac{4}{5}$

b All except **iii**

c Event **iv**

5 **a** Jane/John, Jane/Jack, Jane/Anne, Jane/Dave, Dave/John, Dave/Jack, Dave/Anne, Anne/John, Anne/Jack, Jack/John

b **i** $\frac{1}{10}$ **ii** $\frac{3}{10}$ **iii** $\frac{3}{10}$ **iv** $\frac{7}{10}$

c All except **iii**

d Event **ii**

6 **a** $\frac{3}{8}$ **b** $\frac{1}{8}$

c All except **ii**

d Outcomes overlap

7 $\frac{3}{20}$

8 $\frac{1}{75}$

9 Not mutually exclusive events

10 **a** **i** 0.25 **ii** 0.4 **iii** 0.7

b Events not mutually exclusive

c Man/woman, American man/American woman

d Man/woman

11 **a** **i** 0.95

ii 0.9 (assuming person chooses one or other)

iii 0.3

b Events not mutually exclusive

c Possible answer: pork and vegetarian

12 These are not mutually exclusive events.

Exercise 13C

1 25

2 1000

3 **a** 260 **b** 40 **c** 130 **d** 10

4 5

5 **a** 150 **b** 100 **c** 250 **d** 0

6 **a** 167 **b** 833

7 1050

8 **a**

Score	1	2	3	4	5	6
Expected occurrences	10	10	10	10	10	10

b 1 × 10 + 2 × 10 + 3 × 10 + 4 × 10 + 5 × 10 + 6 × 10 = 210 = 3.5

c Find the average of the scores, which is $\frac{21}{6}$ = 3.5

9 **a** 0.111 **b** 40

10 281 days

11 Multiply the number of tomato plants by 0.997

12 400

Exercise 13D

1 **a** 23 **b** 20% **c** $\frac{4}{25}$ **d** 480

2 **a** 10 **b** 7 **c** 14% **d** 15%

3 **a**

	1	**2**	**3**	**4**
5	6	7	8	9
6	7	8	9	10
7	8	9	10	11
8	9	10	11	12

b 4

c **i** $\frac{1}{4}$ **ii** $\frac{3}{16}$ **iii** $\frac{1}{4}$

4 **a** 16 **b** 16 **c** 73 **d** $\frac{51}{73}$

5 **a**

1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6
2	4	6	8	10	12

b 3 **c** $\frac{1}{4}$

6 **a** $\frac{2}{45}$ **b** 40% **c** 45%

d No, as you don't know how much the people who get over £350 actually earn

7 $\frac{22}{36} = \frac{11}{18}$

8 **a**

Score of second spinner						
10	10	11	13	15	17	19
8	8	9	11	13	15	17
6	6	7	9	11	13	15
4	4	5	7	9	11	13
2	2	3	5	7	9	11
0	0	1	3	5	7	9
	0	**1**	**3**	**5**	**7**	**9**
	Score of first spinner					

b 9 or 11 **c** 0

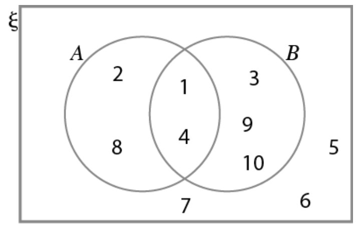
d $\frac{15}{36} = \frac{5}{12}$ **e** $\frac{30}{36} = \frac{5}{6}$

Exercise 13E

1 **a** 0.9 **b** 0.7

2 **a** 0.75 **b** 0.45

3 **a**



b **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{1}{2}$

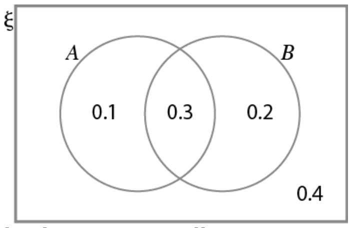
iv $\frac{1}{2}$ **v** $\frac{7}{10}$ **vi** $\frac{1}{5}$

4 **a** **i** 0.52 **ii** 0.48 **iii** 0.65

iv 0.35 **v** 0.82 **vi** 0.35

b 0.3

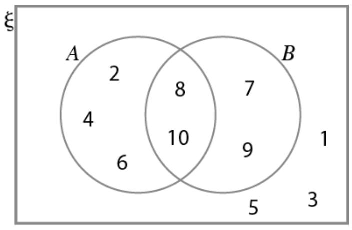
5 a



b i 0.5 **ii** 0.6 **iii** 0.3

6 a 65 **b** 70 **c** 90

7



a $\frac{1}{2}$ **b** $\frac{2}{5}$

c $\frac{7}{10}$ **d** $\frac{1}{5}$

8 a 130

b i $\frac{8}{13}$

ii The probability that a student chosen at random walks to and from school

c $\frac{5}{26}$

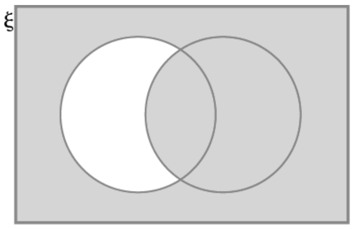
9 0.4

10 0.5

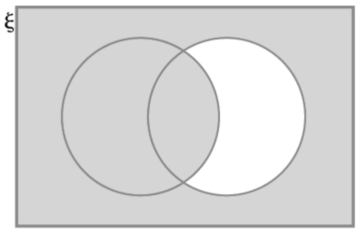
11 a $(A \cup B)'$ **b** $(A \cap B)'$

12 $\frac{37}{80}$

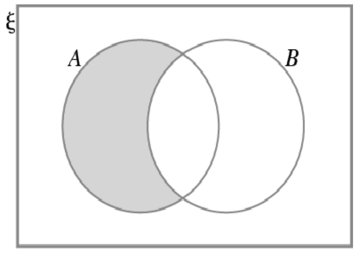
13 a



b



c



Review questions

1 a $\frac{1}{2}$ **b** $\frac{7}{10}$

2 a 0.28

b the frequencies should all be close to 25

3 a i $\frac{3}{10}$ **ii** 10 red, 6 green, 4 blue

b She may not have taken one in the 10 trials

4

	Boys	Girls	Total
Walk to school	9	21	30
Do not walk to school	18	2	20
Total	27	23	50

5 a 110 **b** $\frac{19}{55}$ **c** $\frac{3}{22}$ **d** $\frac{36}{55}$

6 a 0.18 **b** 0.49

c No as the probabilities are close to 0.2

7 a i 0.8 **ii** 12

b No as P(six) = $\frac{1}{6}$, so Tom is likely to throw 10 sixes

8 Draw a two-way table to show the outcomes

+	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

P(score greater than 6) = $\frac{10}{25}$, as $\frac{10}{25} < \frac{1}{2}$, she is likely to lose the game

9 a 0.3 **b** 0.5 **c** 0.6 **d** 0.3

Chapter 14 – Number: Powers and standard form

Exercise 14A

1 a 2^4 **b** 3^5 **c** 7^2 **d** 5^3

e 10^7 **f** 6^4 **g** 4^1 **h** 1^7

i 0.5^4 **j** 100^3

2 a 3 × 3 × 3 × 3

b 9 × 9 × 9

c 6 × 6

d 10 × 10 × 10 × 10 × 10

e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

f 8

g $0.1 \times 0.1 \times 0.1$

h 2.5×2.5

i $0.7 \times 0.7 \times 0.7$

j 1000×1000

3 **a** 16 **b** 243 **c** 49
d 125 **e** 10 000 000 **f** 1296
g 4 **h** 1 **i** 0.0625
j 1 000 000

4 **a** 81 **b** 729 **c** 36
d 100 000 **e** 1024 **f** 8
g 0.001 **h** 6.25 **i** 0.343
j 1 000 000

5 125 m^3

6 **b** 10^2 **c** 2^3 **d** 5^2

7 3: 3, 9, 27, 81, 243, 729 – pattern is 3, 9, 7, 1

4: 4, 16, 64, 256, 1024, 4096 – pattern is 4, 6

5: 5, 125, 625, 3125, 15 625, 78 125 – all 5

8 **a** 1 **b** 4 **c** 1
d 1 **e** 1

9 Any power of 1 is equal to 1.

10 10^6

11 10^6

12 **a** 1 **b** –1 **c** 1 **d** 1 **e** –1

13 **a** 1 **b** –1 **c** –1 **d** 1 **e** 1

14 2^{24}, 4^{12}, 8^8, 16^6

15 $x = 6$

Exercise 14B

1 **a** 5^4 **b** 5^3 **c** 5^2 **d** 5^3 **e** 5^{-5}

2 **a** 6^3 **b** 6^0 **c** 6^6 **d** 6^{-7} **e** 6^2

3 **a** a^3 **b** a^5 **c** a^7
d a^4 **e** a^2 **f** a^1

4 **a** Any two values such that $x + y = 10$

b Any two values such that $x - y = 10$

5 **a** 4^6 **b** 4^{15} **c** 4^6
d 4^{-6} **e** 4^6 **f** 4^0

6 **a** $6a^5$ **b** $9a^2$ **c** $8a^6$
d $-6a^4$ **e** $8a^8$ **f** $-10a^{-3}$

7 **a** $3a$ **b** $4a^3$ **c** $3a^4$
d $6a^{-1}$ **e** $4a^7$ **f** $5a^{-4}$

8 **a** $8a^5b^4$ **b** $10a^3b$ **c** $30a^{-2}b^{-2}$
d $2ab^3$ **e** $8a^{-5}b^7$ **f** $4a^5b^{-5}$

9 **a** $3a^3b^2$ **b** $3a^2c^4$ **c** $8a^2b^2c^3$

10 **a** Possible answer: $6x^2 \times 2y^5$ and $3xy \times 4xy^4$
b Possible answer: $24x^2y^7 \div 2y^2$ and $12x^6y^8 \div x^4y^3$

11 12 ($a = 2$, $b = 1$, $c = 3$)

12 $1 = a^x \div a^x = a^{x-x} = a^0$

Exercise 14C

1 **a** 60 000 **b** 120 000 **c** 150
d 42 000 **e** 1400 **f** 300
g 400 **h** 8000 **i** 160 000
j 4500 **k** 8000 **l** 250 000

2 **a** 5 **b** 50 **c** 25 **d** 30 **e** 7
f 300 **g** 6 **h** 30 **i** 1 **j** 15
k 40 **l** 5 **m** 40 **n** 320

3 **a** 54 400 **b** 16 000

4 $30 \times 90\,000 = 2\,700\,000$
$600 \times 8000 = 4\,800\,000$
$5000 \times 4000 = 20\,000\,000$
$200\,000 \times 700 = 140\,000\,000$

5 1400 million

6 **a** 31 **b** 310 **c** 3100 **d** 31 000

7 **a** 65 **b** 650 **c** 6500 **d** 65 000

8 **a** 0.31 **b** 0.031 **c** 0.0031 **d** 0.000 31

9 **a** 0.65 **b** 0.065 **c** 0.0065 **d** 0.000 65

10 **a** 250 **b** 34.5 **c** 4670
d 346 **e** 207.89 **f** 56 780
g 246 **h** 0.76 **i** 999 000
j 23 456 **k** 98 765.4 **l** 43 230 000
m 345.78 **n** 6000 **o** 56.7
p 560 045

11 **a** 0.025 **b** 0.345
c 0.004 67 **d** 3.46
e 0.207 89 **f** 0.056 78
g 0.0246 **h** 0.0076
i 0.000 000 999 **j** 2.3456
k 0.098 7654 **l** 0.000 043 23
m 0.000000034578 **n** 0.000 000 000 06
o 0.000 0005 67 **p** 0.005 600 45

12 **a** 230 **b** 578 900
c 4790 **d** 57 000 000
e 216 **f** 10 500
g 0.000 32 **h** 9870

13 **a**, **b** and **c**

14 Power 24 means more digits in the answer, so Venus is heavier.

15 6

Exercise 14D

1 **a** 0.31 **b** 0.031 **c** 0.0031 **d** 0.000 31

2 **a** 0.65 **b** 0.065 **c** 0.0065 **d** 0.000 65

3 **a** $9999999999 \times 10^{99}$
b $0.000000001 \times 10^{-99}$(depending on number of digits displayed)

4 **a** 31 **b** 310 **c** 3100 **d** 31 000

5 **a** 65 **b** 650 **c** 6500 **d** 65 000

6 **a** 250 **b** 34.5 **c** 0.004 67
d 34.6 **e** 897 000 **f** 0.00865
g 60 000 000 **h** 0.000 567

7 **a** 2.5×10^2 **b** 3.45×10^{-1}
c 4.67×10^4 **d** 3.4×10^9
e 2.078×10^{10} **f** 5.678×10^{-4}
g 6×10^{-4} **h** 5.67×10^{-3}
i 5.60045×10^1

8 2.81581×10^5, 3×10^1, 1.382101×10^6

9 1.298×10^7, 2.997×10^9, 9.3×10^4

10 100

Exercise 14E

1 **a** 5.67×10^3 **b** 6×10^2
c 3.46×10^{-1} **d** 7×10^3
e 1.6 **f** 2.3×10^7
g 3×10^{-6} **h** 2.56×10^6
i 4.8×10^2 **j** 1.12×10^2
k 6×10^{-1} **l** 2.8×10^6

2 **a** 4.81×10^8 **b** 9.15×10^{12}
c 5.67×10^9 **d** 1.46×10^{14}
e 1.63×10^{22} **f** 1.2×10^9
g 1.08 **h** 6.4×10^2
i 1.2×10^1 **j** 2.88
k 2.5×10^7 **l** 8×10^{-6}

3 **a** 2.64×10^{14} **b** 1.22×10^8
c 1.6×10^9 **d** 3.9×10^{-2}
e 9.6×10^8 **f** 4.6×10^{-7}
g 2.1×10^3 **h** 3.6×10^7
i 1.5×10^2 **j** 3.5×10^9
k 1.6×10^4 **l** 3.81×10^8
k 7.18×10^{12}

4 **a** 2.7×10 **b** 1.6×10^{-2}
c 2×10^{-1} **d** 4×10^{-8}
e 2×10^5 **f** 6×10^{-2}
g 2×10^{-5} **h** 5×10^2
i 2×10

5 **a** 5.4×10 **b** 2.9×10^{-3}
c 1.1 **d** 6.3×10^{-10}
e 2.8×10^2 **f** 5.5×10^{-2}
g 4.9×10^2 **h** 8.6×10^6

6 2×10^{13}, 1×10^{-10}, mass = 2×10^3 g (2 kg)

7 **a** (2^{63}) 9.2×10^{18} grains
b $2^{64} - 1 = 1.8 \times 10^{19}$

8 **a** 6×10^7 sq miles **b** 30%

9 5×10^4

10 2.3×10^5

11 4.55×10^8 kg.

12 **a** 2.048×10^6 **b** 4.816×10^6

13 2.5×10^2

14 9.41×10^4

Review questions

1 **a** **i** 2^4 **ii** 2^8
b **i** 10^3 **iii** 10^9

2 **a** 1 500 000 **b** 6 000 000 000

3 **a** 196
b Units digits is 5 × 5 = 25 so it should end in 5

4 **a** 7^9 **b** x^4
c Adds numbers and multiplies powers, but should be the other way round. $15x^9$

5 **a** t^8 **b** m^5 **c** $9x^6$ **d** $10a^7h^5$

6 **a** x^{11} **b** m^{-5} **c** $8k^5m^3$

7 **a** 7.5×10^4 **b** 0.009

8 **a** $2y$ **b** $8m^9p^{12}$

9 2.48×10^{-7}

10 1000 litres in a cubic metre, $5.3 \times 10^{24} \div 2000 = 2.65 \times 10^{21}$

11 Yes: $(6.5 \times 10^4)^2 = 4.225 \times 10^9$
$(6 \times 10^4)^2 = 3.6 \times 10^9$
$(2.5 \times 10^4)^2 = 6.25 \times 10^8 = 0.625 \times 10^9$
$3.6 \times 10^9 + 0.625 \times 10^9 = 4.225 \times 10^9$

12 $2.6 \times 10^7 \div 2 = 1.3 \times 10^7$,
short side = $1.3 \times 10^7 - 8 \times 10^6 = 13 \times 10^6 - 8 \times 10^6 = 5 \times 10^6$
Area = $5 \times 10^6 \times 8 \times 10^6 = 40 \times 10^{12} = 4 \times 10^{13}$

13 1.5×10^7 sq miles

14 13 875 000

15 Any value from 1×10^8 to 1×10^9 (excluding 1×10^8 and 1×10^9), i.e. any value of the form $a \times 10^8$ where $1 < a < 10$

16 38.625

17 **a** 4.16×10^7 cm **b** 1.056×10^{14} cm^2

Chapter 15 – Algebra: Equations and inequalities

Exercise 15A

1 **a** 30 **b** 21 **c** 72 **d** 12
e 6 **f** $10\frac{1}{2}$ **g** –10 **h** 7
i 11 **j** 2 **k** 7 **l** $2\frac{4}{5}$
m 1 **n** $11\frac{1}{2}$ **o** $\frac{1}{5}$

2 Any valid equations

3 **a** Amanda
b First line: Betsy adds 4 instead of multiplying by 5.
Second line: Betsy adds 5 instead of multiplying by 5.
Fourth line: Betsy subtracts 2 instead of dividing by 2.

4 **a** $\frac{x+10}{5} = 9.50$
b £37.50

5 **a** $\frac{8}{3}$ **b** Student's own checks

Exercise 15B

1 **a** $\frac{1}{2}$ **b** $1\frac{1}{5}$ **c** 2 **d** –2
e –1 **f** –2 **g** –2 **h** –1

2 Any values that work, e.g. $a = 2$, $b = 3$ and $c = 30$.

3 55

4 3 cm

5 5

6 Multiplying out the brackets and simplifying gives $4x - 24 = 0$ which has the solution $x = 6$

7 168°

Exercise 15C

1 **a** $x = 2$ **b** $y = 1$ **c** $a = 7$ **d** $t = 4$
e $p = 2$ **f** $k = -1$ **g** $m = 3$ **h** $s = -2$

2 $3x - 2 = 2x + 5$, $x = 7$

3 **a** $d = 6$ **b** $x = 11$ **c** $y = 1$ **d** $h = 4$
e $b = 9$ **f** $c = 6$

4 **a** $6x + 3 = 6x + 10$; $6x - 6x = 10 - 3$; $0 = 7$, which is obviously false. Both sides have $6x$, which cancels out.
b Multiplying out the brackets gives $12x + 18 = 12x + 18$, which is true for all values of x

5 $8x + 7 + x + 4 = 11x + 5 - x - 4$, $x = 10$

6 **a** They are both equal to the length of the rectangle
b 70 cm^2

7 **a** 15
b –1
c $2(n + 3)$, $2(n + 3) - 5$
d $2(n + 3) - 5 = n$, $2n + 6 - 5 = n$, $2n + 1 = n$, $n = -1$

8 $4x + 18 = 3x + 1 + 50$, $x = 33$. Large bottle 1.5 litres, small bottle 1 litre

9 8

Exercise 15D

1 **a** $x = 4\frac{1}{2}$, $y = 1\frac{1}{2}$ **b** $x = -2$, $y = 4$
c $x = 2\frac{1}{2}$, $y = -1\frac{1}{2}$

2 **a** $a = 7$, $b = 10$ **b** $c = 4$, $d = 11$
c $e = 5$, $f = 3$

3 $x = 12$, $y = 2$

Exercise 15E

1 **a** $x = 9$, $y = -2$
b $x = \frac{1}{2}$, $y = 5$ **c** $x = -3$, $y = -10$

2 **a** $x = 2\frac{1}{4}$, $y = 6\frac{1}{2}$
b $x = 4$, $y = 3$ **c** $x = 5$, $y = 3$

3 **a** $x = 1$, $y = 3$ **b** $x = 5$, $y = 9$

Exercise 15F

1 **a** $x = 2$, $y = 5$ **b** $x = 4$, $y = -3$
c $x = 1$, $y = 7$ **d** $x = \frac{1}{2}$, $y = -\frac{3}{4}$
e $x = -1$, $y = 5$ **f** $x = 1\frac{1}{2}$, $y = \frac{3}{4}$

2 **a** $x = 5$, $y = 1$ **b** $x = 3$, $y = 8$
c $x = 9$, $y = 1$ **d** $x = 7$, $y = 3$
e $x = 4$, $y = 2$ **f** $x = 6$, $y = 5$

Exercise 15G

1 **a** 3 is the first term. The next term is $3 \times a + b$, which equals 14.
b $14a + b = 47$ **c** $a = 3$, $b = 5$
d 146, 443

2 Amul £7.20, Kim £3.50

3 **a** $3t + 5b = 810$, $3t + 3b = 630$
b £10.20

4 **a** They are the same equation. Divide the first by 2 and it is the second, so they have an infinite number of solutions.
b Double the second equation to get $6x + 2y = 14$ and subtract to get $9 = 14$. The left-hand sides are the same if the second is doubled so they cannot have different values.

5 **a** $10x + 5y = 840$, $8x + 10y = 1044$
b £4.07

6 **a** My age minus 6 equals 2 × (my son's age minus 6)
b $x = 46$ and $y = 26$

7 (1, –2) is the solution to equations A and C; (–1, 3) is the solution to equations A and D; (2, 1) is the solution to B and C; (3, –3) is the solution to B and D.

8 84p

9 10.3 kg

10 £4.40

11 $p = 36$, $c = 22$. Total weight for Baz is 428 pounds so he can carry the load safely on his trailer.

12 $b =$ £3.50, $p =$ £1.75. Camilla needs £35 so she will not have enough money.

13 **a** Intersections points are (0, 6), (1, 3) and (2, 4). Area is 2 cm^2.
b Intersection points are (0, 3), (6, 0) and (4, –1). Area is 6 cm^2.

14 When Carmen worked out (2) – (3), she should have got $y = 6$
When Jeff rearranged $2x + 8 - x = 10$, he should have got $x = 2$
They also misunderstood 'two, six' as this means $x = 2$ and $y = 6$, not the other way round.

Exercise 15H

1 **a** $y \leq 3$ **b** $x < 6$ **c** $t \geq 18$
d $x < 7$ **e** $x \leq 3$ **f** $t \geq 5$

2 **a** 16 **b** 3 **c** 7

3 $2x + 3 < 20$, $x < 8.50$, so the most each could cost is £8.49

4 **a** Because 3 + 4 = 7, which is less than the third side of length 8
b $x + x + 2 > 10$, $2x + 2 > 10$, $2x > 8$, $x > 4$, so smallest value of x is 5

5 **a** $x = 6$ and $x < 3$ scores −1 (nothing in common), $x < 3$ and $x > 0$ scores +1 (1 in common for example), $x > 0$ and $x = 2$ scores +1 (2 in common), $x = 2$ and $x \geq 4$ scores −1 (nothing in common), so we get − 1 + 1 + 1 − 1 = 0
b $x > 0$ and $x = 6$ scores +1 (6 in common), $x = 6$ and $x \geq 4$ scores +1 (6 in common), $x \geq 4$ and $x = 2$ scores −1 (nothing in common), $x = 2$ and $x < 3$ scores +1 (2 in common). + 1 + 1 − 1 + 1 = 2
c Any acceptable combination, e.g. $x = 2$, $x < 3$, $x > 0$, $x \geq 4$, $x = 6$

6 **a** $y \leq 4$ **b** $x \geq -2$ **c** $x \leq \frac{14}{5}$
d $x > 38$ **e** $x < 6\frac{1}{2}$ **f** $y \leq \frac{7}{5}$

7 **a** $3 < x < 6$ **b** $2 < x < 5$ **c** $-1 < x \leq 3$
d $1 \leq x < 4$ **e** $2 \leq x < 4$ **f** $0 \leq x \leq 5$

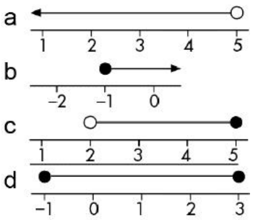
8 **a** {4, 5} **b** {3, 4} **c** {0, 1, 2, 3}
d {1, 2, 3} **e** {2, 3} **f** {0, 1, 2, 3, 4, 5}

9 6

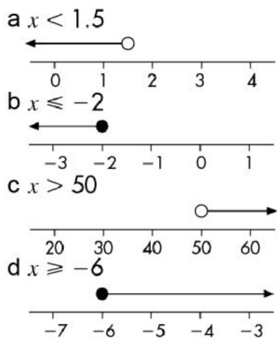
Exercise 15I

1 **a** $x < 2$ **b** $x \geq -1$ **c** $3 < x < 6$

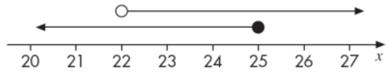
2



3



4 **a** Because 3 apples plus the chocolate bar cost more that £1.20: $x > 22$
b Because 2 apples plus the chocolate bar left Max with at least 16p change: $x \leq 25$
c



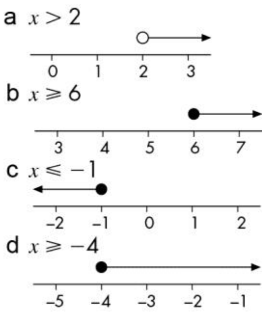
d Apples could cost 23p, 24p or 25p.

5 Any two inequalities that overlap only on the integers −1, 0, 1 and 2 – for example, $x \geq -1$ and $x < 3$

6 1 and 4

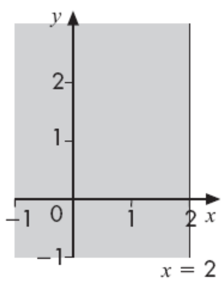
7 $4(35 - 7x) \leq 84$
$35 - 7x \leq 21$
$7x \geq 14$
$x \geq 2$
Also $35 - 7x > 0$
$x < 5$
So the diagram represents this.

8

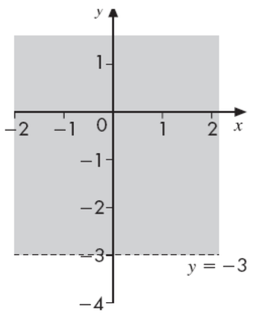


Exercise 15J

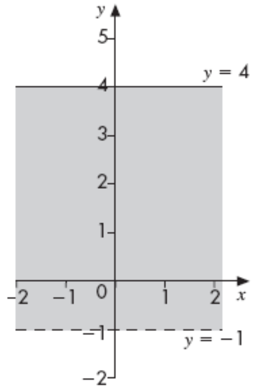
1



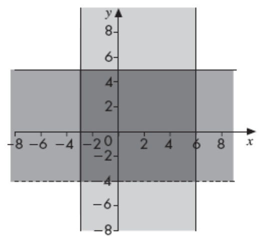
2



3

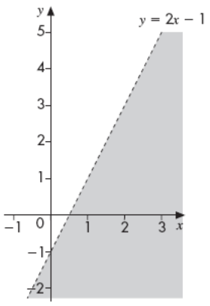


4 a

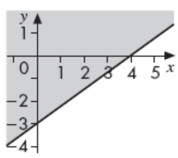


b **i** Yes **ii** Yes **iii** No

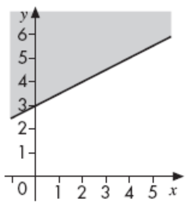
5



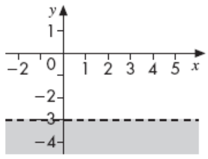
6 a



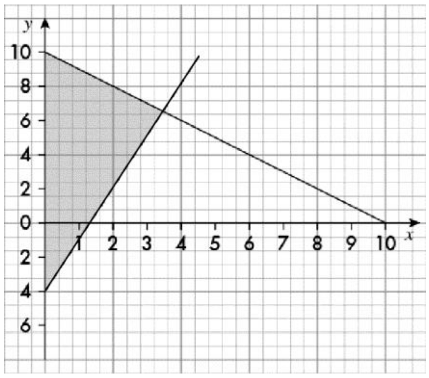
b



c

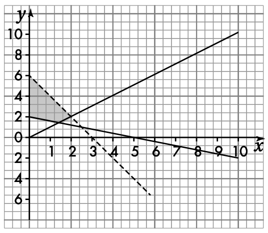


7 a–d



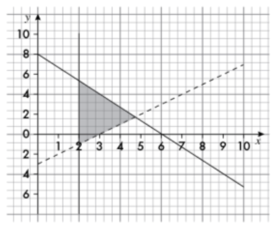
e **i** No **ii** Yes **iii** Yes

8 a–f



g **i** No **ii** No **iii** Yes

9 a



b **i** No **ii** Yes
iii Yes **iv** No

10 For example, $x \geq 1$, $y \leq 3$ and $y \geq x + 1$. There are many other valid answers.

11 May be true: a, c, d, g, h
Must be false: b, e
Must be true: f

12 Test a point such as the origin (0, 0), so 0 < 0 + 2, which is true. So the side that includes the origin is the required side.

13 a (3, 0) **b** (4, 5)

14 £59.50

Exercise 15K

1 a 4 and 5 **b** 4 and 5 **c** 2 and 3

2 $x = 3.5$

3 a $x = 3.7$
b i $x = 2.4$ **ii** $x = 2.8$ **iii** $x = 3.2$
c $x = 5.8$

4 Student's own working

5 $x = 1.50$

6

Guess	$3x^3 + 2x$	Comment
6	660	Too low
7	1043	Too high
6.5	836.875	Too low
6.8	956.896	Too high
6.7	915.689	Too high
6.6	875.688	Too low
6.65	895.538875	Too low

7 a Area = $x(x + 5) = 100$
b Width = 7.8 cm, length = 12.8 cm

8 Volume = $x \times 2x(x + 8) = 500$, $x^3 + 8x^2 = 250$, 4 ⇒ 192, 5 ⇒ 325, 4.4 ⇒ 240.064, 4.5 ⇒ 253.125, 4.45 ⇒ 246.541125, so dimensions are 4.5 cm, 9 cm and 12.5 cm

9 Steph is correct because if 7.05 is too low then the answer will round up to 7.1

10 a Cube is x^3, hole is $\frac{x}{2} \times \frac{x}{2} \times 8 = 2x^2$.
Cube minus hole is 1500
b 12 ⇒ 1440, 13 ⇒ 1859, 12.1 ⇒ 1478.741, 12.2 ⇒ 1518.168, 12.15 ⇒ 1498.368 375 so the value of $x = 12.2$ (to 1 dp)

11 2.76 and 7.24

Review questions

1 8

2 3 years

3 Length is 5.5 m, width is 2.5 m and area is 13.75 m^2. Carpet costs £123.75

4 a B: $\frac{3}{8}x$, C: $\frac{3}{8}x$, D: $\frac{1}{4}x$

b $\frac{3}{8}x = 300$, 800 cars

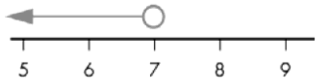
c : $\frac{1}{4}x = 500$, 750 cars

5 No, as $x + x + 2 + x + 4 + x + 6 = 360$ gives $x = 87°$ so the consecutive numbers (87, 89, 91, 93) are not even but odd

6 2 hr 10 min

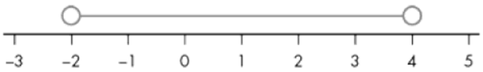
7 –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8

8 a x = 7, **b** x < 7



9 a 6.3 **b** Solve as a linear equation

10 i $-3 < x < 1$, number line b;
ii $-2 < x < 4$, number line below;
iii $-1 < x < 2$, number line a



11 2.78

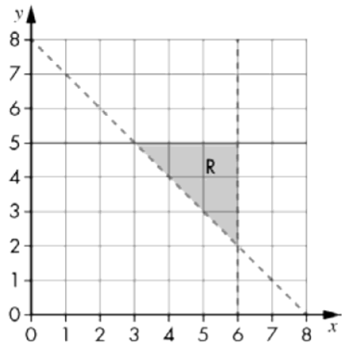
12 £62

13 £195

14 a $x = 4$, $y = 3$
b i $1000x + 1000y = 7000 \rightarrow x + y = 7$
ii $984x - 984y = 984 \rightarrow x - y = 1$
c $a = 9$, $b = 5$

15 Let straight part of track = D, inner radius of end = r, outer radius = $r + x$
x being the width of the track
Length of inner track = $2D + 2\pi r = 300$ **(i)**
Length of outer track = $2D + 2\pi(r + x) = 320$ **(ii)**
Subtract equation **i** from **ii** to give
$2\pi(r + x) - 2\pi r = 20$
$2\pi r + 2\pi x - 2\pi r = 20$
$2\pi x = 20$
$x = 3.2$ (2 s.f.)

16 a



b $4\frac{1}{2}$ square units
c It's infinite

17 –4, –3, –2, –1, 0, 1, 2, 3, 4

18 a $x + y \geq 7$, $y \leq 2x - 1$, $y \geq \frac{1}{2}x$
b $y \leq x - 3$, $x > 2$, $x + y < 8$`

Chapter 16 – Number: Counting, accuracy, powers and surds

Exercise 16A

1 **a** 0.5 **b** $0.\dot{3}$ **c** 0.25
d 0.2 **e** $0.1\dot{6}$ **f** $0.\dot{1}4285\dot{7}$
g 0.125 **h** $0.\dot{1}$ **i** 0.1
j $0.\dot{0}7692\dot{3}$

2 **a** $\frac{4}{7}$ = 0.5714285…
$\frac{5}{7}$ = 0.714 285 7…
$\frac{6}{7}$ = 0.8571428…
b They all contain the same pattern of digits, starting at a different point in the pattern.

3 $0.\dot{1}$, $0.\dot{2}$, $0.\dot{3}$, etc. Digit in decimal fraction same as numerator.

4 $0.\dot{0}\dot{9}$, $0.\dot{1}\dot{8}$, $0.\dot{2}\dot{7}$, etc. Sum of digits in recurring pattern = 9. First digit is one less than numerator.

5 0.444 ..., 0.454 ..., 0.428 ..., 0.409 ..., 0.432 ..., 0.461 ...;
$\frac{9}{22}$, $\frac{3}{7}$, $\frac{16}{37}$, $\frac{4}{9}$, $\frac{5}{11}$, $\frac{6}{13}$

6 **a** $\frac{1}{8}$ **b** $\frac{17}{50}$ **c** $\frac{29}{40}$ **d** $\frac{5}{16}$
e $\frac{89}{100}$ **f** $\frac{1}{20}$ **g** $2\frac{7}{20}$ **h** $\frac{7}{32}$

7 **a** $0.08\dot{3}$ **b** 0.0625 **c** 0.05
d 0.04 **e** 0.02

8 **a** $\frac{4}{3}$ **b** $\frac{6}{5}$ **c** $\frac{5}{2}$
d $\frac{10}{7}$ **e** $\frac{20}{11}$ **f** $\frac{15}{4}$

9 **a** 0.75, $1.\dot{3}$; $0.8\dot{3}$, 1.2; 0.4, 2.5; 0.7, $1.\dot{4}2857\dot{1}$;
0.55, $1.\dot{8}\dot{1}$; $0.2\dot{6}$, 3.75
b Not always true, e.g. reciprocal of 0.4 ($\frac{2}{5}$) is
$\frac{5}{2}$ = 2.5

10 1 ÷ 0 is infinite, so there is no finite answer.

11 **a** 10 **b** 2
c The reciprocal of a reciprocal is always the original number.

12 The reciprocal of *x* is greater than the reciprocal of *y*. For example, 2, 10, reciprocal of 2 is 0.5, reciprocal of 10 is 0.1, and 0.5 > 0.1

13 Possible answer: $-\frac{1}{2} \times -2 = 1$, $-\frac{1}{3} \times -3 = 1$

14 **a** 24.24242 ... **b** 24
c $\frac{24}{99} = \frac{8}{33}$

15 **a** $\frac{8}{9}$ **b** $\frac{34}{99}$ **c** $\frac{5}{11}$ **d** $\frac{21}{37}$
e $\frac{4}{9}$ **f** $\frac{2}{45}$ **g** $\frac{13}{90}$ **h** $\frac{1}{22}$
i $2\frac{7}{9}$ **j** $7\frac{7}{11}$ **k** $3\frac{1}{3}$ **l** $2\frac{2}{33}$

16 **a** true **b** true **c** recurring

17 **a** $\frac{9}{9}$ **b** $\frac{45}{90} = \frac{1}{2} = 0.5$

Exercise 16B

1 **a** 14 **b** 100 **c** 5 **d** 13

2 8, 27 and 25

3 13 and 14

4 5 and 6

5 Answers can be about the same as these
a **i** $\sqrt{(66 \times 100)} \approx 8.1 \times 10 = 81$
ii $\sqrt{49} = 7$, so $\sqrt{45} \approx 6.7$
iii $\sqrt[3]{64} = 4$, $\sqrt[3]{27} = 3$, so $\sqrt[3]{40} \approx 3.4$
iv $5.8^4 \approx 6^4 = 36 \times 36 \approx 30 \times 40 = 1200$
v $\sqrt[3]{45\,000} = \sqrt[3]{45} \times 10 \approx 35$
b **i** 81.24 **ii** 6.708 **iii** 3.42
iv 1132 **v** 35.57

Exercise 16C

1 **a** $\frac{1}{5^3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10^5}$ **d** $\frac{1}{3^2}$
e $\frac{1}{8^2}$ **f** $\frac{1}{9}$ **g** $\frac{1}{w^2}$ **h** $\frac{1}{t}$
i $\frac{1}{x^m}$ **j** $\frac{4}{m^3}$

2 **a** 3^{-2} **b** 5^{-1} **c** 10^{-3} **d** m^{-1} **e** t^{-n}

3 **a** **i** 2^4 **ii** 2^{-1} **iii** 2^{-4} **iv** -2^3
b **i** 10^3 **ii** 10^{-1} **iii** 10^{-2} **iv** 10^6
c **i** 5^3 **ii** 5^{-1} **iii** 5^{-2} **iv** 5^{-4}
d **i** 3^2 **ii** 3^{-3} **iii** 3^{-4} **iv** -3^5

4 **a** $\frac{5}{x^3}$ **b** $\frac{6}{t}$ **c** $\frac{7}{m^2}$ **d** $\frac{4}{q^4}$
e $\frac{10}{y^5}$ **f** $\frac{1}{2x^3}$ **g** $\frac{1}{2m}$ **h** $\frac{3}{4t^4}$
i $\frac{4}{5y^3}$ **j** $\frac{7}{8x^5}$

5 **a** $7x^{-3}$ **b** $10p^{-1}$ **c** $5t^{-2}$
d $8m^{-5}$ **e** $3y^{-1}$

6 **a** **i** 25 **ii** $\frac{1}{125}$ **iii** $\frac{4}{5}$
b **i** 64 **ii** $\frac{1}{16}$ **iii** $\frac{5}{256}$
c **i** 8
ii $\frac{1}{32}$
iii $\frac{9}{2}$ or $4\frac{1}{2}$
d **i** 1 000 000
ii $\frac{1}{1000}$
iii $\frac{1}{4}$

7 24 (32 – 8)

8 *x* = 8 and *y* = 4 (or *x* = *y* = 1)

9 $\frac{1}{2097152}$

10 **a** x^{-5}, x^0, x^5 **b** x^5, x^0, x^{-5} **c** x^5, x^{-5}, x^0

Exercise 16D

1 **a** 5 **b** 25 **c** 3 **d** 5 **e** 20 **f** 5 **g** 3 **h** 10 **i** 3 **j** 2 **k** $\frac{1}{4}$ **l** $\frac{1}{2}$ **m** $\frac{1}{3}$ **n** $\frac{1}{5}$ **o** $\frac{1}{10}$

2 **a** $\frac{5}{6}$ **b** $1\frac{2}{3}$ **c** $\frac{8}{9}$ **d** $1\frac{4}{5}$ **e** $\frac{5}{8}$ **f** $\frac{3}{5}$ **g** $\frac{1}{4}$ **h** $2\frac{1}{2}$ **i** $\frac{4}{5}$ **j** $1\frac{1}{7}$

3 $\left(x^{\frac{1}{n}}\right)^n = x^{\frac{1}{n}\times n} = x^1 = x$, but

$\left(\sqrt[n]{x}\right)^n = \sqrt[n]{x} \times \sqrt[n]{x}$ … n times $= x$, so

$x^{\frac{1}{n}} = \sqrt[n]{x}$

4 $64^{-\frac{1}{2}} = \frac{1}{8}$, others are both $\frac{1}{2}$

5 Possible answer: The negative power gives the reciprocal, so $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$
The power one-third means cube root, so you need the cube root of 27 which is 3, so $27^{\frac{1}{3}} = 3$ and $\frac{1}{27^{\frac{1}{3}}} = \frac{1}{3}$

6 Possible answers include $x = 16$ and $y = 64$, $x = 25$ and $y = 125$

Exercise 16E

1 **a** 16 **b** 25 **c** 216 **d** 81

2 **a** $t^{\frac{2}{3}}$ **b** $m^{\frac{3}{4}}$ **c** $k^{\frac{2}{5}}$ **d** $x^{\frac{3}{2}}$

3 **a** 4 **b** 9 **c** 64 **d** 3125

4 **a** $\frac{1}{5}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$ **e** $\frac{1}{4}$ **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $\frac{1}{3}$

5 **a** $\frac{1}{125}$ **b** $\frac{1}{216}$ **c** $\frac{1}{8}$ **d** $\frac{1}{27}$ **e** $\frac{1}{256}$ **f** $\frac{1}{4}$ **g** $\frac{1}{4}$ **h** $\frac{1}{9}$

6 **a** $\frac{1}{100\,000}$ **b** $\frac{1}{12}$ **c** $\frac{1}{25}$ **d** $\frac{1}{27}$ **e** $\frac{1}{32}$ **f** $\frac{1}{32}$ **g** $\frac{1}{81}$ **h** $\frac{1}{13}$

7 $8^{-\frac{2}{3}} = \frac{1}{4}$, others are both $\frac{1}{8}$

8 Possible answer: The negative power gives the reciprocal, so $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}}$
The power one-third means cube root, so we need the cube root of 27 which is 3 and the power 2 means square, so $3^2 = 9$, so $27^{\frac{2}{3}} = 9$ and $\frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$

9 $3 = x^{-\frac{2}{3}} \div x^{-1}$, $3 = x^{\frac{1}{3}}$, $x = 27$

Exercise 16F

1 **a** $\sqrt{6}$ **b** $\sqrt{15}$ **c** 2 **d** 4 **e** $2\sqrt{10}$ **f** 3 **g** $2\sqrt{3}$ **h** $\sqrt{21}$ **i** $\sqrt{14}$ **j** 6 **k** 6 **l** $\sqrt{30}$

2 **a** 2 **b** $\sqrt{5}$ **c** $\sqrt{6}$ **d** $\sqrt{3}$ **e** $\sqrt{5}$ **f** 1 **g** $\sqrt{3}$ **h** $\sqrt{7}$ **i** 2 **j** $\sqrt{6}$ **k** 1 **l** 3

3 **a** $2\sqrt{3}$ **b** 15 **c** $4\sqrt{2}$ **d** $4\sqrt{3}$ **e** $8\sqrt{5}$ **f** $3\sqrt{3}$ **g** 24 **h** $3\sqrt{7}$ **i** $2\sqrt{7}$ **j** $6\sqrt{5}$ **k** $6\sqrt{3}$ **l** 30

4 **a** $\sqrt{3}$ **b** 1 **c** $2\sqrt{2}$ **d** $\sqrt{2}$ **e** $\sqrt{5}$ **f** $\sqrt{3}$ **g** $\sqrt{2}$ **h** $\sqrt{7}$ **i** $\sqrt{7}$ **j** $2\sqrt{3}$ **k** $2\sqrt{3}$ **l** 1

5 **a** a **b** 1 **c** $\sqrt{a}$

6 **a** $3\sqrt{2}$ **b** $2\sqrt{6}$ **c** $2\sqrt{3}$ **d** $5\sqrt{2}$ **e** $2\sqrt{2}$ **f** $3\sqrt{3}$ **g** $4\sqrt{3}$ **h** $5\sqrt{3}$ **i** $3\sqrt{5}$ **j** $3\sqrt{7}$ **k** $4\sqrt{2}$ **l** $10\sqrt{2}$ **m** $10\sqrt{10}$ **n** $5\sqrt{10}$ **o** $7\sqrt{2}$ **p** $9\sqrt{3}$

7 **a** 36 **b** $16\sqrt{30}$ **c** 54 **d** 32 **e** $48\sqrt{6}$ **f** $48\sqrt{6}$ **g** $18\sqrt{15}$ **h** 84 **i** 64 **j** 100 **k** 50 **l** 56

8 **a** $20\sqrt{6}$ **b** $6\sqrt{15}$ **c** 24 **d** 16 **e** $12\sqrt{10}$ **f** 18 **g** $20\sqrt{3}$ **h** $10\sqrt{21}$ **i** $6\sqrt{21}$ **j** 36 **k** 24 **l** $12\sqrt{30}$

9 **a** 6 **b** $3\sqrt{5}$ **c** $6\sqrt{6}$ **d** $2\sqrt{3}$ **e** $4\sqrt{5}$ **f** 5 **g** $7\sqrt{3}$ **h** $2\sqrt{7}$ **i** 6 **j** $2\sqrt{7}$ **k** 5 **l** Does not simplify

10 **a** $2\sqrt{3}$ **b** 4 **c** $6\sqrt{2}$ **d** $4\sqrt{2}$ **e** $6\sqrt{5}$ **f** $24\sqrt{3}$ **g** $3\sqrt{2}$ **h** $\sqrt{7}$ **i** $10\sqrt{7}$ **j** $8\sqrt{3}$ **k** $10\sqrt{3}$ **l** 6

11 **a** abc **b** $\frac{a}{c}$ **c** $c\sqrt{b}$

12 **a** 20 **b** 24 **c** 10 **d** 24 **e** 3 **f** 6

13 **a** $\frac{3}{4}$ **b** $8\frac{1}{3}$ **c** $\frac{5}{16}$ **d** 12 **e** 2

14 **a** False **b** False

15 Possible answer: $\sqrt{3} \times 2\sqrt{3}$ (= 6)

16 $(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a+b})^2$, $a + 2\sqrt{ab} + b = a + b$,
Cancel a and b, $2\sqrt{ab} = 0$, so $a = 0$ and/or $b = 0$.

Exercise 16G

1 Expand the brackets each time.

2 **a** $2\sqrt{3} - 3$ **b** $3\sqrt{2} - 8$ **c** $10 + 4\sqrt{5}$
d $12\sqrt{7} - 42$ **e** $15\sqrt{2} - 24$ **f** $9 - \sqrt{3}$

3 **a** $2\sqrt{3}$ **b** $1 + \sqrt{5}$ **c** $-1 - \sqrt{2}$
d $\sqrt{7} - 30$ **e** -41 **f** $7 + 3\sqrt{6}$
g $9 + 4\sqrt{5}$ **h** $3 - 2\sqrt{2}$ **i** $11 + 6\sqrt{2}$

4 **a** $3\sqrt{2}$ cm **b** $2\sqrt{3}$ cm **c** $2\sqrt{10}$ cm

5 **a** $\sqrt{3} - 1$ cm^2 **b** $2\sqrt{5} + 5\sqrt{2}$ cm^2
c $2\sqrt{3} + 18$ cm^2

6 **a** $\frac{\sqrt{3}}{3}$ **b** $\frac{\sqrt{2}}{2}$ **c** $\frac{\sqrt{5}}{5}$ **d** $\frac{\sqrt{3}}{6}$
e $\sqrt{3}$ **f** $\frac{5\sqrt{2}}{2}$ **g** $\frac{3}{2}$ **h** $\frac{5\sqrt{2}}{2}$
i $\frac{\sqrt{21}}{3}$ **j** $\frac{\sqrt{2}+2}{2}$ **k** $\frac{2\sqrt{3}-3}{3}$ **l** $\frac{5\sqrt{3}+6}{3}$

7 **a** **i** 1 **ii** –4 **iii** 2
iv 17 **v** –44
b They become whole numbers. Difference of two squares makes the 'middle terms' (and surds) disappear.

8 **a** Possible answer: $\sqrt{2}$ and $\sqrt{2}$ or $\sqrt{2}$ and $\sqrt{8}$
b Possible answer: $\sqrt{2}$ and $\sqrt{3}$

9 **a** Possible answer: $\sqrt{2}$ and $\sqrt{2}$ or $\sqrt{8}$ and $\sqrt{2}$
b Possible answer: $\sqrt{3}$ and $\sqrt{2}$

10 Possible answer: $80^2 = 6400$, so $80 = \sqrt{6400}$ and $10\sqrt{70} = \sqrt{7000}$
Since 6400 < 7000, there is not enough cable.

11 $9 + 6\sqrt{2} + 2 - (1 - 2\sqrt{8} + 8) = 11 - 9 + 6\sqrt{2} + 4\sqrt{2} = 2 + 10\sqrt{2}$

12 $x^2 - y^2 = (1 + \sqrt{2})^2 - (1 - \sqrt{8})^2 = 1 + 2\sqrt{2} + 2 - (1 - 2\sqrt{8} + 8) = 3 - 9 + 2\sqrt{2} + 4\sqrt{2} = -6 + 6\sqrt{2}$
$(x + y)(x - y) = (2 - \sqrt{2})(3\sqrt{2}) = 6\sqrt{2} - 6$

13 $4\sqrt{2} - (\sqrt{2} - 1) = 3\sqrt{2} + 1$. $(\sqrt{2} - 1)(3\sqrt{2} + 1) = 5 - 2\sqrt{2}$

14 **a** **i** $3 + n\sqrt{2}$ **ii** $\frac{n+1}{\sqrt{3}}$
b **i** $\left(\sqrt{5}\right)^n$ **ii** $\left(5 + \sqrt{2}\right) \times \left(\sqrt{2}\right)^{n-1}$

Exercise 16H

1 **a** 6.5 cm ≤ 7 cm < 7.5 cm
b 115 g ≤ 120 g < 125 g
c 3350 km ≤ 3400 km < 3450 km
d 49.5 mph ≤ 50 mph < 50.5 mph
e £5.50 ≤ £6 < £6.50
f 16.75 cm ≤ 16.8 cm < 16.85 cm
g 15.5 kg ≤ 16 kg < 16.5 kg
h 14 450 people ≤ 14 500 people < 14 550 people
i 54.5 miles ≤ 55 miles < 55.5 miles
j 52.5 miles ≤ 55 miles < 57.5 miles

2 **a** 5.5 cm ≤ 6 cm < 6.5 cm
b 16.5 kg ≤ 17 kg < 17.5 kg
c 31.5 min ≤ 32 min < 32.5 min
d 237.5 km ≤ 238 km < 238.5 km
e 7.25 m ≤ 7.3 m < 7.35 m
f 25.75 kg ≤ 25.8 kg < 25.85 kg
g 3.35 h ≤ 3.4 h < 3.45 h
h 86.5 g ≤ 87 g < 87.5 g
i 4.225 mm ≤ 4.23 mm < 4.235 mm
j 2.185 kg ≤ 2.19 kg < 2.195 kg
k 12.665 min ≤ 12.67 min < 12.675 min
l 24.5 m ≤ 25 m < 25.5 m
m 35 cm ≤ 40 cm < 45 cm
n 595 g ≤ 600 g < 605 g
o 25 min ≤ 30 min < 35 min
p 995 m ≤ 1000 m < 1050 m
q 3.95 m ≤ 4.0 m < 4.05 m
r 7.035 kg ≤ 7.04 kg < 7.045 kg
s 11.95 s ≤ 12.0 s < 12.05 s
t 6.995 m ≤ 7.00 m < 7.005 m

3 **a** 7.5 m, 8.5 m **b** 25.5 kg, 26.5 kg
c 24.5 min, 25.5 min **d** 84.5 g, 85.5 g
e 2.395 m, 2.405 m **f** 0.15 kg, 0.25 kg
g 0.055 s, 0.065 s **h** 250 g, 350 g
i 0.65 m, 0.75 m **j** 365.5 g, 366.5 g
k 165 weeks, 175 weeks **l** 205 g, 215 g

4 There are 16 empty seats and the number getting on the bus is from 15 to 24 so it is possible if 15 or 16 get on.

5 C: The chain and distance are both any value between 29.5 and 30.5 metres, so there is no way of knowing if the chain is longer or shorter than the distance.

6 2 kg 450 grams

7 **a** < 65.5 g **b** 64.5 g
c < 2620 g **d** 2580 g

8 345, 346, 347, 348, 349

9 Any number in range $4 < a < 5$, e.g. 4.5

Exercise 16I

1 Minimum 65 kg, maximum 75 kg

2 Minimum is 19, maximum is 20

3 **a** 12.5 kg **b** 20

4 3 years 364 days (Jack is on his fifth birthday; Jill is 9 years old tomorrow)

5 **a** 38.25 cm^2 ≤ area < 52.25 cm^2
b 37.1575 cm^2 ≤ area < 38.4475 cm^2
c 135.625 cm^2 ≤ area < 145.225 cm^2

6 **a** 5.5 m ≤ length < 6.5 m, 3.5 m ≤ width < 4.5 m
b 29.25 m^2
c 18 m

7 79.75 m^2 ≤ area < 100.75 m^2

8 216.125 cm^3 ≤ volume < 354.375 cm^3

9 12.5 metres

10 Yes, because they could be walking at 4.5 mph and 2.5 mph meaning that they would cover 4.5 miles + 2.5 miles = 7 miles in 1 hour

11 20.9 m ≤ length < 22.9 m (3 sf)

12 16.4 cm^2 ≤ area < 21.7 cm^2 (3 sf)

13 **a** **i** 64.1 cm^3 ≤ volume < 69.6 cm^3 (3 sf)
ii £22 578 ≤ price < £24 515 (nearest £)
b 23 643 ≤ price < £23 661 (nearest £)
c Errors in length compounded by being used 3 times in **a**, but errors in weight only used once in **b**

14 **a** 14.65 s ≤ time < 14.75 s
b 99.5 m ≤ length < 100.5 m
c 6.86 m/s (3 sf)

15 **a** 1.25% (3 sf)
b 1.89% (3 sf)

16 3.41 cm ≤ length < 3.43 cm (3 sf)

17 5.80 cm ≤ length < 5.90 cm (3 sf)

18 14 s ≤ time < 30 s

19 Cannot be certain as limits of accuracy for all three springs overlap:
Red: 12.5 newtons to 13.1 newtons
Green: 11.8 newtons to 13.2 newtons
Blue: 9.5 newtons to 12.9 newtons
For example, all tensions could be 12 newtons

Exercise 16J

1 Number of possible permutations is 7! ÷ 2!5! = 21. Of these any pair of the first 5 coins will be less than a £1, which is 5! ÷ 3! 2! = 10. Hence 11 pairs will have a value greater than £1.

2 6, 16, etc. up to 196, which is 19 plus 60 up to 69, which is 9 (66 already counted) plus 160 up to 169 which is 9 (166 already counted) giving a total of 37

3 **a** **i** 5040 **ii** 2.43×10^{18} (3 sf)
b This depends on your calculator but 69! = 1.71×10^{98}, which is about the number of atoms in QUINTILLION (look it up) universes.

4 **a** 10^4 = 10 000 **b** 13^4 = 28 561

5 3×13^3 = 6591

6 **a** 504 **b** 495

7 **a** 30 **b** 56

8 **a** 10^3 = 1000 **b** 6

9 8 × 7 × 6 = 336

10 **a** 16 ways of choosing an Ace followed by a King out of 52 × 52 ways of picking 2 cards with replacement, so $\frac{16}{2704} = \frac{1}{169}$
b Still 16 ways of taking an ace followed by a King but out of 52 × 51 so $\frac{16}{2652} = \frac{4}{663}$

11 **a** 1 6 15 20 15 6 1, 1 7 21 35 35 21 7 1, 1 8 28 56 70 56 28 8 1, 1 9 36 84 126 126 84 36 9 1, 1 10 45 120 210 252 210 120 45 10 1
b **i** 10 **ii** 1 **iii** 28 **iv** 1
c ${}_5C_2$ is the 3rd value in the 6th row, ${}_8C_6$ is the 7th value in the 9th row
d **i** 20 **ii** 8 **iii** 3 **iv** 70
e 1

12 **a** 31 **b** 8 (2^8 = 256)

13 **a** C 435 **b** B 48 **c** B 12 **d** D 12
e C 455 **f** A 64 = 1296
g B 5! = 120
h A 6^5 = 7776
i A $26^2 \times 10^2$ = 67 600
j B 9 × 5 = 45
k A 10! = 3 628 800
l Mixture of D and C 60

14 12

15 144

16 6 (RBB, RBY, RYY, RRB, RRY, RRR)

17 Assume 5 seat car is full. Driver is fixed, other 4 seats can be filled by 6×5×4×3 arrangements = 360. In the 4 seat car driver is fixed but the 2 people left can sit in 3 × 2 = 6 arrangements. Total 6 × 360 = 2160. Now assume 4 seat car is full. That is 6 × 5 × 4 arrangements = 120 and in the 5 seat car the 3 people left can sit in 4 × 3 × 2 arrangements = 24, 24 × 120 = 2880. That is a total of 5040 different seating arrangements.

18 1000

19 105

20 **a** 5! × 3! = 720 **b** 5! × 3 = 360

21 **a** **i** 79 380 **ii** 35 280
iii 52 920
b Players could only be assigned to their own group. If not, then the number of possible teams would increase.

22 **a** 40 320
b 109 600

23 They are both correct and give the answer 511.

Review questions

1 **a** 13 **b** 10 **c** 13

2 **a** 1845 **b** 1854

3 8, 16 and 36

4 19

5 ${}_{12}P_2$ = 132, ${}_9C_4$ = 126 so ${}_{12}P_2$ is greater

6 **a** 12^4 = 20 736 **b** 3×12^3 = 5184

7 **a** 5 × 4 × 3 × 2 × 1 = 5! = 120
b There will be 24 starting with each letter and CODES will be the first CO word so 13th in the list

8 **a** 3^4 = 81 **b** 8 **c** –3

9 **a** $\frac{1}{25}$ **b** 6^4 = 1296

10 $\frac{7}{15}$

11 $a = 7$, $b = -1$

12 $6\sqrt{2}$

13 a $x = 0.5454..$, $100x = 54.5454..$, $99x = 54$

$x = \frac{54}{99}$, cancel by 9

b $0.35454.. = 0.3 + 0.05454.. = \frac{3}{10} + \frac{6}{110} =$

$\frac{33}{110} + \frac{6}{110} = \frac{39}{110}$

14 $\frac{11}{45}$

15 a 9 **b** $5\sqrt{2}$

16 a $\frac{\sqrt{5}}{5}$ **b** 2

17 a i $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

ii $14 + 4\sqrt{6}$

b $2^2 + (2 + \sqrt{6})^2 = 4 + 4 + 6 + 4\sqrt{6} = 14 + 4\sqrt{6} = (\sqrt{2} + \sqrt{12})^2$ so the sides obey Pythagoras' theorem

18 a $\sqrt{27} = 5.20$ m

b Cube side 2.95 m has diagonal 5.07 m. Max length pole is 5.005 m so it will fit round the corner.

Chapter 17 – Algebra: Quadratic Equations

Exercise 17A

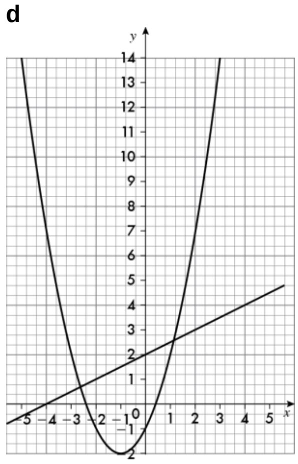
1 a Values of y: 27, 16, 7, 0, −5, −8, −9, −8, −5, 0, 7

b −8.8 **c** 3.4 or −1.4

2 a Values of y: 2, −1, −2, −1, 2, 7, 14

b 0.25 **c** 0.7 or −2.7

d



e (1.1, 2.6) and (−2.6, 0.7)

3 a Values of y: 15, 9, 4, 0, −3, −5, −6, −6, −5, −3, 0, 4, 9

b −0.5 and 3

4 a Same answer

b

x	–3	–2	–1	0	1	2	3	4	5	6	7
y	28	19	12	7	4	3	4	7	12	19	28

Since the quadratic graph has a vertical line of symmetry and the y-values for $x = 1$ and $x = 3$ are the same, this means that the y-values will be symmetric about $x = 2$. Hence the y-values will be the same for $x = 0$ and $x = 4$, and so on.

5 Points plotted and joined should give a parabola.

6 Line A has a constant in front, so is 'thinner' than the rest.

Line B has a negative in front, so is 'upside down'.

Line C does not pass through the origin.

Exercise 17B

1 a –2, –5 **b** 4, 9 **c** –6, 3

2 a –4, –1 **b** 2, 4 **c** –2, 5 **d** –3, 5 **e** –6, 3 **f** –1, 2 **g** –5 **h** 7

3 $x(x + 40) = 48\,000$, $x^2 + 40x - 48\,000 = 0$,

$(x + 240)(x - 200) = 0$

Fence is 2 × 200 + 2 × 240 = 880 m

4 a –10, 3 **b** –4, 11 **c** –8, 9 **d** 8, 9 **e** 1 **f** –6, 7 **g** –2, 3

5 Mario was correct.

Sylvan did not make it into a standard quadratic and only factorised the x terms. She also incorrectly solved the equation $x - 3 = 4$.

6 40 cm

7 48 km/h

8 a 4, 9

b i 2, –2, 3, –3 **ii** 16, 81

iii 5, 6, 10, 11

Exercise 17C

1 a $\frac{1}{3}$, –3 **b** $1\frac{1}{3}$, $-\frac{1}{2}$ **c** $-\frac{1}{5}$, 2

d $-2\frac{1}{2}$, $3\frac{1}{2}$ **e** $-\frac{1}{6}$, $-\frac{1}{3}$ **f** $\frac{2}{3}$, 4

g $\frac{1}{2}$, –3 **h** $\frac{5}{2}$, $-\frac{7}{6}$ **i** $-1\frac{2}{3}$, $1\frac{2}{5}$

j $1\frac{3}{4}$, $1\frac{2}{7}$ **k** $\frac{2}{3}$, $\frac{1}{8}$ **l** $\pm\frac{1}{4}$

m $-2\frac{1}{4}$, 0 **n** $\pm 1\frac{2}{5}$ **o** $-\frac{1}{3}$, 3

2 a 7, –6 **b** $-2\frac{1}{2}$, $1\frac{1}{2}$ **c** –1, $\frac{11}{13}$

d $-\frac{2}{5}$, $\frac{1}{2}$ **e** $-\frac{1}{3}$, $-\frac{1}{2}$ **f** $\frac{1}{5}$, –2

g 4 **h** –2, $\frac{1}{8}$ **i** $-\frac{1}{3}$, 0

j ±5 **k** $-1\frac{2}{3}$ **l** $\pm 3\frac{1}{2}$

m $-2\frac{1}{2}$, 3

3 a Both only have one solution: $x = 1$.

b B is a linear equation, but A and C are quadratic equations.

4 a $(5x - 1)^2 = (2x + 3)^2 + (x + 1)^2$, when expanded and collected into the general quadratic, gives the required equation.

b $(10x + 3)(2x - 3)$, $x = 1.5$; area = 7.5 cm^2.

5 **a** Show by substituting into the equation
b $-\frac{24}{5}$

6 5, 0.5

7 Area = 22.75, width = 3.5 m

Exercise 17D

1 **a** 1.77, –2.27 **b** 3.70, –2.70
c –0.19, –1.53 **d** –1.39, –2.27
e 1.37, –4.37 **f** 0.44, –1.69
g 1.64, 0.61 **h** 0.36, –0.79
i 1.89, 0.11

2 13

3 $x^2 - 3x - 7 = 0$

4 Terry gets $x = \frac{4+\sqrt{0}}{8}$ and June gets $(2x - 1)^2 = 0$, which only give one solution $x = \frac{1}{2}$

5 6.54, 0.46

6 1.25, 0.8

7 **a** **i** –0.382, –2.618 **ii** 6.414, 3.586
iii 7.531, –0.531 **iv** 1.123, –7.123

b Since a = 1, answers are $\frac{-b+\sqrt{b^2-4c}}{2}$ and $\frac{-b-\sqrt{b^2-4c}}{2}$. When added,

$$\frac{-b+\sqrt{b^2-4c}-b-\sqrt{b^2-4c}}{2}=\frac{-b-b}{2}=\frac{-2b}{2}=-b$$

Exercise 17E

1 **a** 52 (TWO) **b** 0 (ONE)
c –23 (NONE) **d** –7 (NONE)
e 68 (TWO) **f** –35 (NONE)
g –4 (NONE) **h** 0 (ONE)
I 409 (TWO)

2 300

3 $x^2 + 3x - 1 = 0$; $x^2 - 3x - 1 = 0$; $x^2 + x - 3 = 0$; $x^2 - x - 3 = 0$

4 2 or –10

5 Can be factorised: $b^2 - 4ac$ = 1849, 1, 49, 1024, 900
Cannot be factorised: $b^2 - 4ac$ = 41, 265, 3529, 216, 76
For those that can be factorised, $b^2 - 4ac$ is a square number

Exercise 17F

1 **a** $(x - 2)^2 - 4$ **b** $(x + 7)^2 - 49$
c $(x - 3)^2 - 9$ **d** $(x + 3)^2 - 9$
e $(x - 5)^2 - 25$ **f** $(x + 10)^2 - 100$
g $(x - 2)^2 - 5$ **h** $(x + 3)^2 - 6$
i $(x + 4)^2 - 22$ **j** $(x + 1)^2 - 2$
k $(x - 1)^2 - 8$ **l** $(x + 9)^2 - 11$

2 **a** 4th, 1st, 2nd and 3rd – in that order
b Write $x^2 - 4x - 3 = 0$ as $(x - 2)^2 - 7 = 0$, Add 7 to both sides, square root both sides, Add 2 to both sides
c **i** $x = -3 \pm \sqrt{2}$ **ii** $x = 2 \pm \sqrt{7}$

3 **a** $-2 \pm \sqrt{5}$ **b** $-7 \pm 3\sqrt{6}$
c $3 \pm \sqrt{6}$ **d** $5 \pm \sqrt{30}$
e $-10 \pm \sqrt{101}$ **f** $-4 \pm \sqrt{22}$

4 **a** 1.45, –3.45 **b** 5.32, –1.32
c –4.16, 2.16

5 Check for correct proof.

6 $p = -14$, $q = -3$

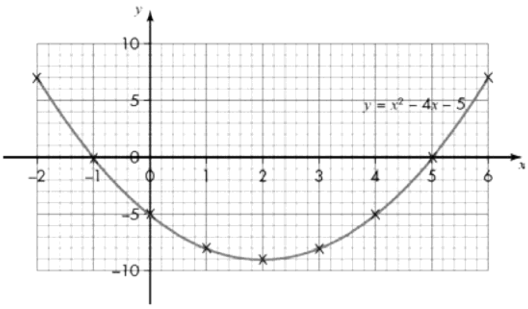
7 **a** $x^2 - 12x + 40 = (x - 6)^2 + 4 \geq 4$ for all x
b Doesn't intersect the x-axis

8 The answers are 42, – 58. The equation can be factorised as $(x - 42)(x + 58) = 0$ but it would be hard to find the factors of 2436. Completing the square works well because $x^2 + 16x - 2436 = (x + 8)^2 - 2500$ and you can find the square root of 2500 without a calculator. Completing the square is therefore the better of the two non-calculator methods. The formula could also be used without a calculator because $b^2 - 4ac$ = 10 000 so the square root can be taken, but you would have to work out $16^2 + 4 \times 2436$ in order to get there.

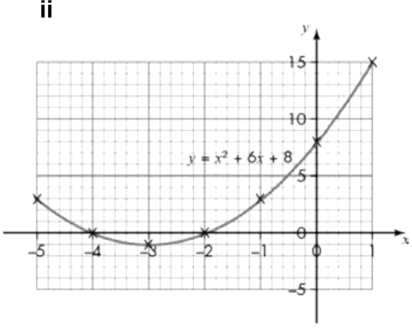
9 H, C, B, E, D, J, A, F, G, I

Exercise 17G

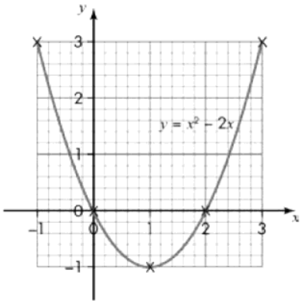
a **i**



ii



iii



b Each equation is written in the form $x^2 + ax + b$. You should find that the y-intercept is the value of b. Graph (i) has its y-intercept at (0, –5), graph (ii) has its y-intercept at (0, 8) and graph (iii) has its y-intercept at (0, 0). Note that the graph (iii)'s equation has no value for b, so $b = 0$.

c **i** $x = 5$ or –1 **ii** $x = –2$ or –4
iii $x = 0$ or 2

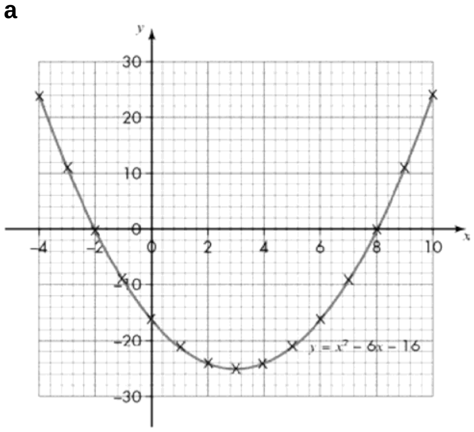
d The two x-intercepts have a product of b and add up to $–a$. This works because the x-intercepts are the answers of the quadratic equations when $y = 0$.

e The value of x for the turning point is exactly halfway between the values of x for the x-intercepts. By completing the square, you should also be able to see that the x co-ordinate is the value that makes the brackets zero and the y co-ordinate is the value at the end.

Exercise 17H

1 **a** **i** (0, –3) **ii** (–1, 0) and (3, 0)
iii (1, –4)
b **i** (0, 5) **ii** (–5, 0) and (1, 0)
iii (–2, 9)

2
a



b **i** (0, –16) **ii** (–2, 0) and (8, 0)
iii (3, –25)

3 **a** roots: (–2, 0) and (2, 0); y-intercept (0, –4)
b roots: (0, 0) and (6, 0); y-intercept (0, 0)
c roots: (–1, 0) and (3, 0); y-intercept (0, –3)
d roots: (–11, 0) and (–3, 0); y-intercept (0, 33)

4 (3, –6)

5 –14

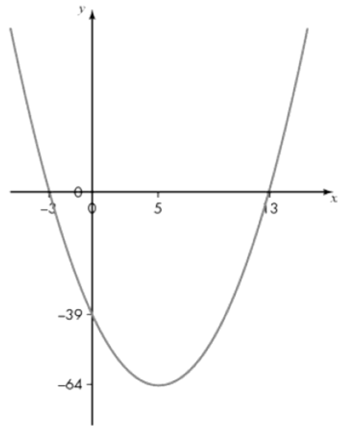
6 –5

7 **a** (2, 0) **b** 2 is the only root

8 roots: (–0.5, 0) and (5, 0); y-intercept (0, –5); turning point: (2.25, –15.125)

9 roots: (4.65, 0) and (7.85, 0); turning point: (6.25, –5.13)

10



11 $y = (x - 3)^2 - 7$, $y = x^2 - 6x + 9 - 7$, $y = x^2 - 6x + 2$

12 **a** (−2, −7)
b **i** $(a, 2b - a^2)$ **ii** $(2a, b - 4a^2)$

13 $y = 2x^2 + 16x + 14$

14 **a** 60 m **b** 80 m, 2 s **c** 6 s

Exercise 17I

1 **a** (0.7, 0.7), (–2.7, –2.7)
b (6, 12), (–1, –2)
c (4, –3), (–3, 4)
d (0.8, 1.8), (–1.8, –0.8)
e (4.6, 8.2), (0.4, –0.2)
f (3, 6), (–2, 1)
g (4.8, 6.6), (0.2, –2.6)
h (2.6, 1.6), (–1.6, –2.6)

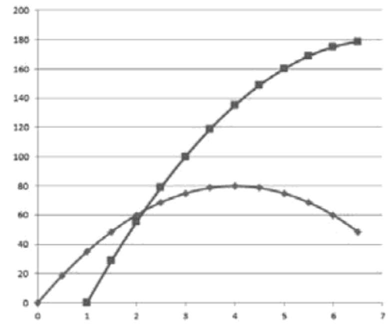
2 **a** (1, 0)
b Only one intersection point
c $x^2 + x(3 – 5) + (–4 + 5) = 0$
d $(x – 1)^2 = 0 \Rightarrow x = 1$
e Only one solution as line is a tangent to curve.

3 **a** There is no solution.
b The graphs do not intersect.
c $x^2 + x + 4 = 0$
d $b^2 – 4ac = –15$
e No solution as the discriminant is negative and there is no square root of a negative number.

4 **a** $x = 4$, $y = 31$
b There is only one solution because the graphs have the same shape and are at a constant distance apart.

5 a Proof

b



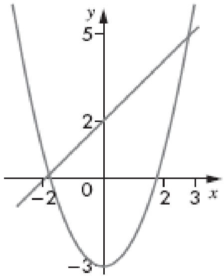
c 2.17 seconds

Exercise 17J

1 a i –1.4, 4.4 **ii** –2, 5 **iii** –0.6, 3.6
b 2.6, 0.4

2 a i –1.6, 2.6 **ii** 1.4, –1.4
b i 2.3, –2.3 **ii** 2, –2

3



a 2.2, –2.2 **b** –1.8, 2.8

4 –3.8, 1.8

5 a C and D **b** A and D
c $x^2 + 4x - 1 = 0$ **d** (–1.5, –10.25)

6 a i $y = 5$ **ii** $y = x + 3$
iii $y = -10$ **iv** $y = x$
v $y = 3x - 9$ **vi** $y = 2 - x$
vii $y = -3x$
b $y = \frac{1}{2}x + 3$

7 a i $5 - 5x - x^2 = 0$ **ii** $11 - 6x - x^2 = 0$
iii $9 - 4x - x^2 = 0$ **iv** $30 - 16x - 3x^2 = 0$
b Equation would be $-5 - 4x - x^2 = 0$. $b^2 - 4ac = -4$. Negative $b^2 - 4ac$ has no solutions.

8 a $(x + 2)(x - 1) = 0$ **b** 5 – –2 = +7, not –7
c $y = 2x + 7$

Exercise 17K

1 a (5, –1) **b** (4, 1) **c** (8, –1)

2 a (2, 5) and (–2, –3) **b** (–1, –2) and (4, 3)
c (3, 3) and (1, –1)

3 a (1, 2) and (–2, –1)
b (–4, 1) and (–2, 2)

4 a (3, 4) and (4, 3)
b (0, 3) and (–3, 0)

5 a (3, 2) and (–2, 3) **b** √26

6 a Proof
b $x = -\frac{1}{5}$, $y = -\frac{43}{5}$ or $x = 5$, $y = 7$

7 a Proof
b $x = 4$, $y = -13$ or $x = 8$, $y = 11$

8 a $x = 6$, $y = 7$ or $x = -2$, $y = -9$
b $x = -1$, $y = 2$ or $x = -2$, $y = -1$
c $x = 3$, $y = -5$ or $x = 5$, $y = 3$
d $x = 1$, $y = -8$ or $x = 4$, $y = 7$

9 a (1, 0)
b **iii** as the straight line just touches the curve

10 a (–2, 1)
b i (2, 1) **ii** (–2, –1) **iii** (2, –1)

11 a (2, 4) **b** (1, 0)
c The line is a tangent to the curve.

12 16 m by 14 m

13 30 km/h

14 10p

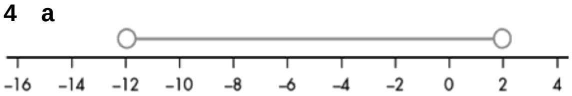
Exercise 17L

1 a $x < -4$, $x > 4$ **b** $-10 \le x \le 10$
c $0 < x < 1$ **d** $x \le -5$, $x \ge 0$
e $-23 < x < 23$ **f** $x \le -\frac{3}{2}$, $x \ge \frac{3}{2}$
g $x < 0$, $x > \frac{8}{3}$ **h** $-\frac{19}{2} \le x \le 0$

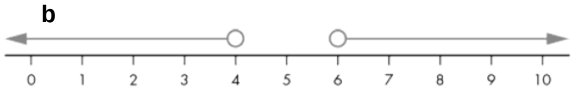
2 a {–3, –2, –1, 0, 1, 2, 3}
b {3, 4, 5, 6}

3 a $x < -2$, $x > 5$ **b** $-7 < x < -5$
c $x \le 1$, $x \ge 5$ **d** $-8 \le x \le 9$
e $\frac{1}{3} \le x \le 3$ **f** $x < -\frac{11}{2}$, $x > -1$
g $x \le \frac{3}{5}$, $x \ge 2$ **h** $-\frac{3}{2} < x < \frac{2}{3}$

4 a



b

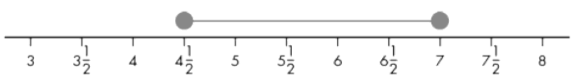


5 a $3 < x \le 6\frac{1}{2}$ **b** $4 < x \le 5$

6 $x < 2$, $x > 10$

7 $x < -\frac{3}{2}$, $x > 5$

8



9 a $-692 < x < 708$
b $x < -4 - \sqrt{5}$, $x > -4 + \sqrt{5}$
c $-0.84 \le x \le 1.44$

10 £288, £364

11 $x < -4$, $-1 < x < 1$, $x > 4$

12 a $\frac{30}{-13} = -2.31 > -6$ and $\frac{30}{2} = 15 > 9$
b $x < -7$, $3 < x < 6$

Review questions

1 a 9 **b** 5

2 a Two **b** One **c** None

3 b –5.27, 1.67

4 b 3.18

5 15 m, 20 m

6 b i –0.3, 3.3 **ii** 0.6, 3.4

7 a (0, 36) **b** (2, 0), (18, 0)
c (10, –64)

8 (1, 7), (7, 1)

9 a –6 **b** 3

10 a $x^2 - 3x - 550 = 0$ **b** 25

11 a $x < -35$, $x > 45$ **b** $-298 < x < 302$
c $x \le -589$, $x \ge 611$

12 2.54 m, 3.54 m

13 210 cm^2

14 (6, 8), (0, –10)

15 a $(p + q)(p - q)$
b $30^2 - 1^2 = (30 + 1)(30 - 1)$
c 3600
d –31, 29

16 0.75 m

17 a $48 - (x - 6)^2$ **b** 48

18 Complete the square –113, 87

19 a, b (1, 4), (5, 20) **c** $x \le 1$, $x \ge 5$

20 $x^2 - 8x + 19 = (x - 4)^2 + 3$
Because $(x - 4)^2$ is a squared term, the smallest possible value it can have is zero.
Hence 3 is the smallest possible value of $(x - 4)^2 + 3$, so $x^2 - 8x + 19$ is always positive.

Chapter 18 – Statistics: Sampling and more complex diagrams

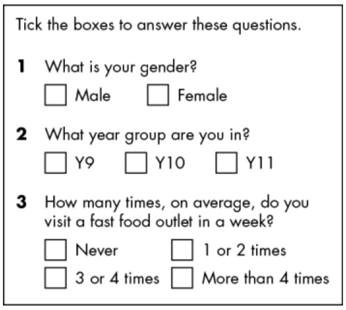
Exercise 18A

1 a Secondary data
b Primary data
c Primary or secondary data
d Primary or secondary data
e Primary data
f Primary or secondary data

2 *Plan the data collection*. Choose a random sample of 30 boys and 30 girls from Year 11.
Collect the data. Ask each student to spell the same 10 words. This will avoid bias. Pick words that are often misspelt, e.g. accommodation, necessary
Choose the best way to process and represent the data. Calculate the mean number and range for the number of correct spellings for the boys and for the girls. Draw a dual bar chart to illustrate the data.
Interpret the data and make conclusions. Compare the mean and range to arrive at a conclusion. Is there a clear conclusion or do you need to change any of the 10 words or take a larger sample?

3 *Plan the data collection*. Choose a random sample of 20 boys and 20 girls from Year 11.
Collect the data. Ask each student, on average, how many hours of sport they play and how many hours of TV they watch each week.
Choose the best way to process and represent the data. Calculate the mean number of hours for the number of hours playing sport and the number of hours watching TV. Draw a scatter diagram to illustrate the data.
Interpret the data and make conclusions. Compare the means and write down the type and strength of correlation for the scatter diagram to arrive at a conclusion. Is there a clear conclusion or do you need to take a larger sample?

4 a e.g.


Tick the boxes to answer these questions.
1 What is your gender?
☐ Male ☐ Female
2 What year group are you in?
☐ Y9 ☐ Y10 ☐ Y11
3 How many times, on average, do you visit a fast food outlet in a week?
☐ Never ☐ 1 or 2 times
☐ 3 or 4 times ☐ More than 4 times

b e.g.

	Boys	Girls
Y9	20	20
Y10	20	20
Y11	20	20

c e.g. Get a list of the names of the students in alphabetical order for each group. Then choose a random sample for each one by picking every 10th student or use random digits on a calculator.

5 248 boys and 310 girls

6 Find the approximate number of men, women, boys and girls in the crowd and then decide on a sample size. A suitable sample size here is 100. Work out the proportion of men in the whole group and work out same proportion in the sample size to give the number of men in the sample. Similarly work out the proportion of women, boys and girls.

7 a There are more students in Year 12 and a different number of boys and girls in both years.

b e.g.

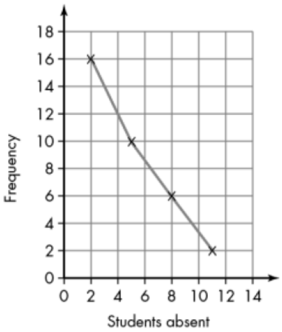
Year group	Boys	Girls	Total
12	21	24	45
13	20	15	35
			80

8 e.g.

	Male	Female	Total
Full time	130	70	200
Part time	40	60	100
			300

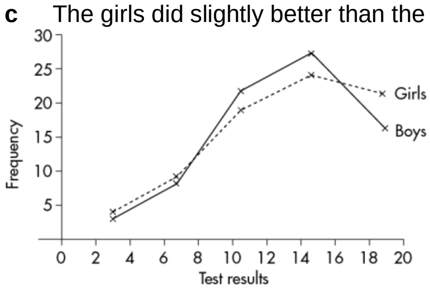
Exercise 18B

1 a–b 4

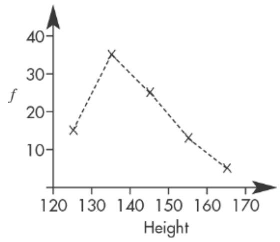


2 **a–b** Boys 12.9, girls 13.1

c The girls did slightly better than the boys

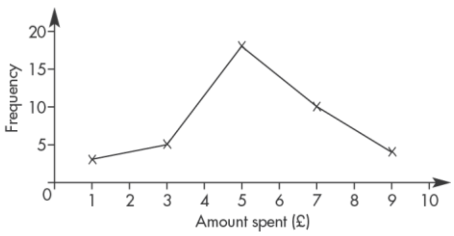


3 **a–b** 140.4 cm



4 **a** **i** 17, 13, 6, 3,1 **ii** £1.45

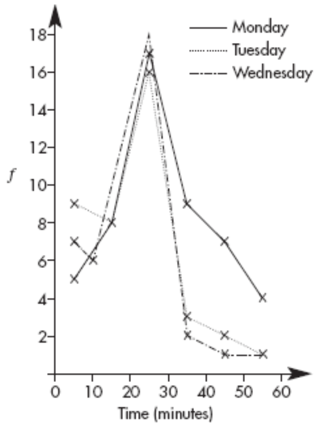
b **i**



ii £5.35

c Much higher mean. Early morning, people just want a paper or a few sweets. Later people are buying food for the day.

5 **a**



b Monday 28.4 min, Tuesday 20.9 min, Wednesday 21.3 min

c There are more patients on a Monday, and so longer waiting times, as the surgery is closed during the weekend.

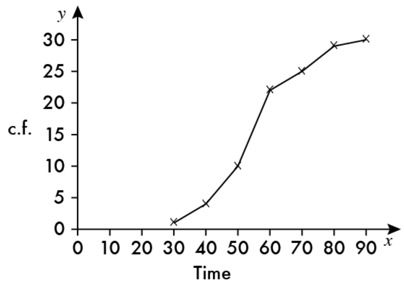
6 2.19 hours

7 That is the middle value of the time group 0 to 1 minute. It would be very unusual for most of them to be exactly in the middle at 30 seconds.

Exercise 18C

1 **a** Cumulative frequencies 1, 4, 10, 22, 25, 28, 30

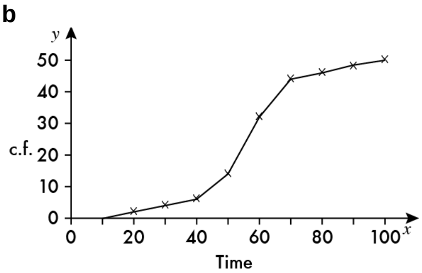
b



c m = 54 s and IQR = 16 s

2 **a** Cumulative frequencies 1, 3, 5, 14, 31, 44, 47 49, 50

b

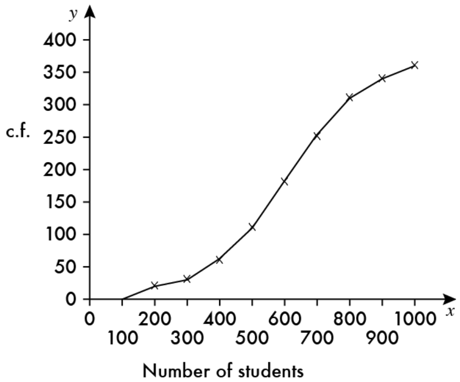


c m = 56 s and IQR = 17 s

d Pensioners as the median is closer to 1 minute and the IQRs are almost the same

3 **a** Cumulative frequencies 12, 30, 63, 113, 176, 250, 314, 349, 360

b



c m = 606 students

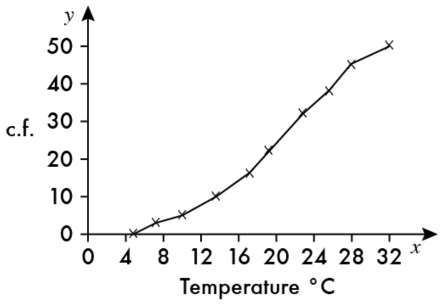
d Q_1 = 455, Q_3 = 732 and IQR = 277

e Approximately 13%

4 **a** Cumulative frequency 2, 5, 10, 16, 22, 31, 39, 45, 50

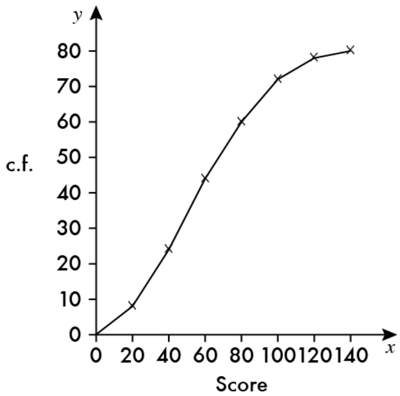
b Because the temperature was recorded to the nearest degree, so for example the highest temperature in the first group could have been 7.5°

c



d m = 20.5 °C and IQR = 10 °C

5 **a**



b m = 56, Q_1 = 37 and Q_3 = 100

c Approximately 18%

6 **a** Paper A m = 66, Paper B m = 56

b Paper A IQR = 25, Paper B IQR = 18

c Paper B is the harder paper, it has a lower median and a lower upper quartile.

d **i** Paper A 43, Paper B 45

ii Paper A 78, Paper B 67

7 Create a grouped frequency table:

Time, t, (minutes)	**Cumulative frequency**	**Frequency, f**	**Mid-point, x**	**$x \times f$**
$0 < t \le 5$	6	6	2.5	15
$5 < t \le 10$	34	28	7.5	210
$10 < t \le 15$	56	22	12.5	275
$15 < t \le 20$	60	4	17.5	70
	Total	60		570

mean = $\frac{570}{60}$ = 9.5 minutes

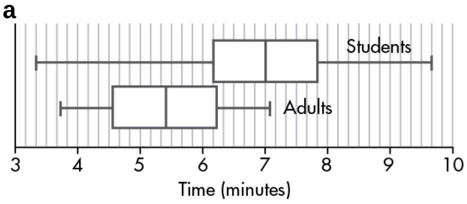
8 Create a grouped frequency table:

Age, a, (years)	**Cumulative frequency**	**Frequency, f**	**Mid-point, x**	**$x \times f$**
$0 < a \le 20$	30	30	10	300
$20 < a \le 40$	95	65	30	1950
$40 < a \le 60$	150	55	50	2750
$60 < a \le 80$	185	35	70	2450
$80 < a \le 100$	200	15	90	1350
	Total	200		8800

mean = $\frac{8800}{200}$ = 44 years

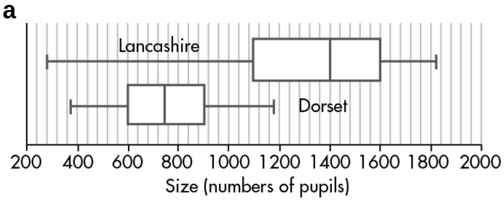
Exercise 18D

1 **a**



b The adults are much quicker than the students. Both distributions have the same interquartile range, but the range is smaller for the adults showing that they are more consistent. The students' median and upper quartiles are 1 minute, 35 seconds higher. The fastest person to complete the calculations was a student, but so was the slowest.

2 **a**



b Schools are much larger in Lancashire than in Dorset since it has a greater median. The interquartile range in Dorset is smaller, showing that they have a more consistent size.

3 **a** The resorts have similar median temperatures, but Resort A has a smaller interquartile range, showing that the temperatures are more consistent. Resort B has a much wider temperature range, where the greatest extremes of temperature are recorded.

b Resort A is probably a better choice as the weather seems more consistent.

4 **a**

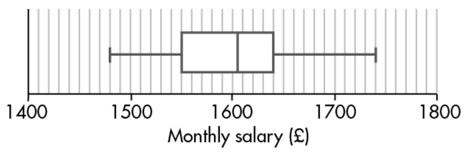
b Both distributions have a similar interquartile range, and there is little difference between the upper quartile values. Men have a wider range of salaries and the men have a higher median. This indicates that the men are better paid than the women.

5 **a**

b m = £1605

c Q_1 = £1550 and Q_3 = £1640

d



6 **a** **i** 24 min **ii** 12 min **iii** 42 min

b **i** 6 min **ii** 17 min **iii** 9 min

c Either doctor with a plausible reason, e.g. Dr Excel because on average, her waiting times are always shorter or Dr Collins because he takes more time with each patient as the interquartile range is more spread out.

7 Many possible answers but not including any numerical values: eg Bude had a higher median amount of sunshine. Bude had a smaller interquartile range, showing more consistent sunshine in Bude. So overall this indicates that Bude had more sunshine on any one day.

8 Create a grouped frequency table using the quartiles:

For the boys

Mark, *m*	**Cumulative frequency**	**Frequency, *f***	**Mid-point, *x***	***x* × *f***
39 < *m* ≤ 65	25	25	52	1300
65 < *m* ≤ 78	50	25	71.5	1787.5
78 < *m* ≤ 87	75	25	82.5	2062.5
87 < *m* ≤ 112	100	25	99.5	2487.5
	Total	100		7637.5

mean = $\frac{7637.5}{100}$ = 76.4 marks (1 dp)

For the girls

Mark, *m*	**Cumulative frequency**	**Frequency, *f***	**Mid-point, *x***	***x* × *f***
49 < *m* ≤ 69	25	25	59	1475
69 < *m* ≤ 78	50	25	73.5	1837.5
78 < *m* ≤ 91	75	25	84.5	2112.5
91 < *m* ≤ 106	100	25	98.5	2462.5
	Total	100		7887.5

mean = $\frac{7887.5}{100}$ = 78.9 marks (1 dp)

The mean is 2.5 marks higher for the girls

Exercise 18E

1 The respective frequency densities on which each histogram should be based are:

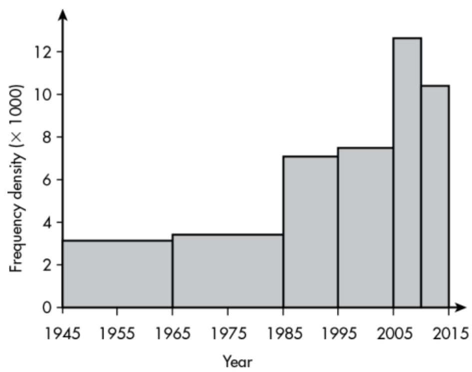
a 2.5, 6.5, 9, 2, 1.5 **b** 3, 6, 10, 4.5

2 The respective frequency densities on which each histogram should be based are:

a 7, 12, 10, 5 **b** 0.4, 1.2, 2.8, 1

c 9, 12, 13.5, 9

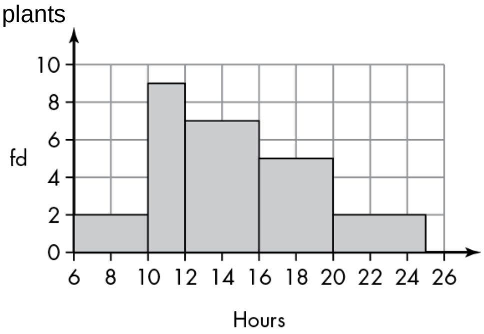
3



4 **a** **i** Work out the class width × frequency density for each bar and add these together, i.e. 5 × 25 + 5 × 30 + 10 × 20 + 10 × 10 + 20 × 5 + 10 × 10

ii 775

b 400

5 **a–b** 

c 14 kg **d** 14.6 kg **e** 33 plants

6 **a**

Speed, *v* (mph)	0 < *v* ≤ 40	40 < *v* ≤ 50	50 < *v* ≤ 60	60 < *v* ≤ 70	70 < *v* ≤ 80	80 < *v* ≤ 100
Frequency	80	10	40	110	60	60

b 360 **c** 64.5 mph **d** 59.2 mph

7 **a** 100 **b** 32.5 **c** 101.5

d 10% of 300 = 30, so the pass mark will be in the 70–80 interval. There are 60 students in this interval and 30 is half of 60. So the pass mark is half way between 70 and 80 = 75

8 **a**

Temperature, *t* (°C)	10 < *t* ≤ 11	11 < *t* ≤ 12	12 < *t* ≤ 14	14 < *t* ≤ 16	16 < *t* ≤ 19	19 < *t* ≤ 21
Frequency	15	15	50	40	45	15

b 12–14 °C **c** 14.5 °C

d 12.6 °C, 17 °C, 4.4 °C **e** 14.8 °C

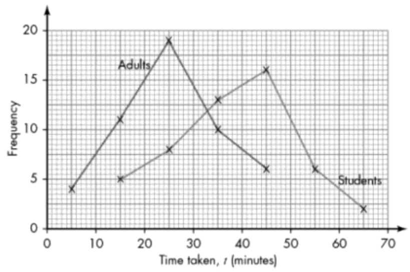
9 0.45

Review questions

1 Choose a suitable sample size and decide whether to use a random sample or a stratified sample. Make sure that the sample is reliable and unbiased.

Remember that the greater the accuracy required, the larger the sample size needs to be. But the larger the sample size, the higher the cost will be and the time taken. Therefore, the benefit of achieving high accuracy in a sample will always have to be set against the cost of achieving it.

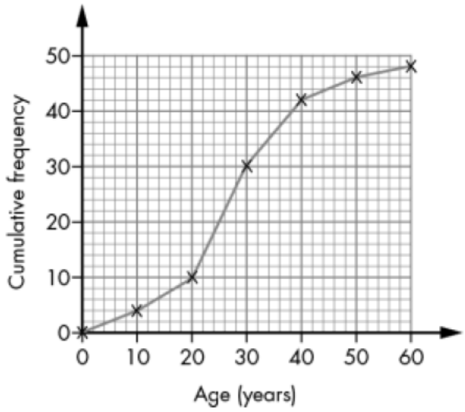
2 **a**



b The adults completed the puzzle quicker as their average time was better. Also their range was smaller which makes them more consistent.

3 **a** Cumulative frequencies: 4, 10, 20, 42, 46, 48

b



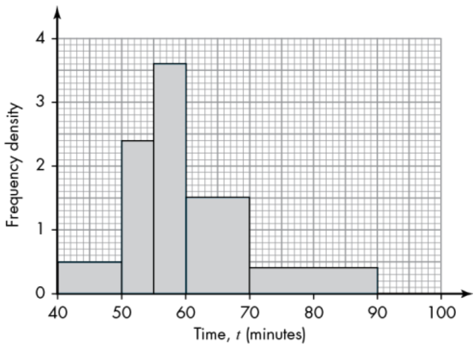
c 32 **d** Q_1 = 22, Q_3 = 37 and IQR = 15

4 **a** **i** £7200 **ii** £6400

b **i** £6000 **ii** £4700

c On average the men's wages are higher as their median is greater. The women's wages are more consistent as their interquartile range is smaller.

5



6 **a**

Age, *t* (years)	9 < *t* ≤ 10	10 < *t* ≤ 12	12 < *t* ≤ 14	14 < *t* ≤ 17	17 < *t* ≤ 19	19 < *t* ≤ 20
Frequency	4	12	8	9	5	1

b 10–12 **c** 13

d 11, 16, 5 **e** 13.4

7 Create a grouped frequency table using the quartiles:

Amount, *m* (£)	**Cumulative frequency**	**Frequency, *f***	**Mid-point, *x***	***x* × *f***
0.50 < *m* ≤ 2.00	20	20	1.25	25
2.00 < *m* ≤ 3.00	40	20	2.50	50
3.00 < *m* ≤ 4.00	60	20	3.50	70
4.00 < *m* ≤ 6.00	80	20	5.00	100
	Total	80		245

mean = $\frac{245}{80}$ = £3.06

8 e.g.

Year 7	Year 8	Year 9	Year 10	Year 11
32	31	36	40	41

Chapter 19 – Probability: Combined events

Exercise 19A

1 **a** $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

2 **a** $\frac{2}{11}$ **b** $\frac{4}{11}$ **c** $\frac{6}{11}$

3 **a** $\frac{1}{3}$ **b** $\frac{2}{5}$ **c** $\frac{11}{15}$ **d** $\frac{11}{15}$ **e** $\frac{1}{3}$

4 **a** 60 **b** $\frac{4}{5}$

5 **a** 0.8 **b** 0.2

6 **a** **i** 0.75 **ii** 0.6 **iii** 0.5 **iv** 0.6
b **i** Cannot add P(red) and P(1) as events are not mutually exclusive
ii 0.75 (= 1 – P(blue))

7 0.46

8 Probabilities cannot be summed in this way as events are not mutually exclusive.

9 **a** **i** 0.4 **ii** 0.5 **iii** 0.9
b 0.45
c 2 hours 12 minutes

10 $\frac{5}{52}$ or 0.096 to 3 decimal places

Exercise 19B

1 **a** 7
b 2, 12
c

Score	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

d **i** $\frac{1}{12}$ **ii** $\frac{5}{9}$ **iii** $\frac{1}{2}$
iv $\frac{7}{36}$ **v** $\frac{5}{12}$ **vi** $\frac{5}{18}$

2 **a** $\frac{1}{12}$ **b** $\frac{11}{36}$ **c** $\frac{1}{6}$ **d** $\frac{5}{9}$

3 **a** $\frac{1}{36}$ **b** $\frac{11}{36}$ **c** $\frac{5}{18}$

4 **a**

Score on second dice						
6	5	4	3	2	1	0
5	4	3	2	1	0	1
4	3	2	1	0	1	2
3	2	1	0	1	2	3
2	1	0	1	2	3	4
1	0	1	2	3	4	5
	1	2	3	4	5	6

Score on first dice

b **i** $\frac{5}{18}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{9}$
iv 0 **v** $\frac{1}{2}$

5 **a** **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$ **iv** $\frac{1}{4}$
b All possibilities are included

6 **a** $\frac{1}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{6}$

7 **a**

Score on second spinner					
5	6	7	8	9	10
4	5	6	7	8	9
3	4	5	6	7	8
2	3	4	5	6	7
1	2	3	4	5	6
	1	2	3	4	5

Score on first spinner

b 6

c **i** $\frac{4}{25}$ **ii** $\frac{13}{25}$ **iii** $\frac{1}{5}$ **iv** $\frac{3}{5}$

8 **a** HHH, HHT, HTH, THH, HTT, THT, TTH, TTT
b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

9 **a** 16 **b** 32 **c** 1024 **d** 2^n

10 **a**

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

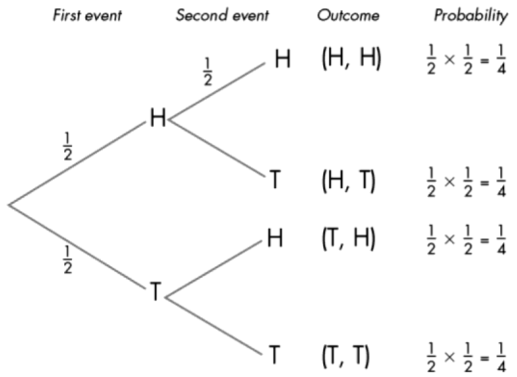
b $\frac{1}{18}$ **c** 18 **d** Twice

11 $\frac{1}{2}$

12 You would need a 3D diagram or there would be too many different events to list.

Exercise 19C

1 a



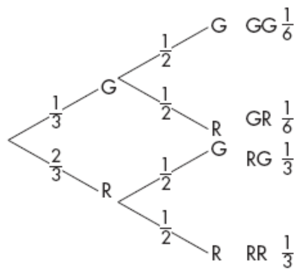
b i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

2 a $\frac{2}{13}$ **b** $\frac{11}{13}$

c i $\frac{1}{169}$ **ii** $\frac{25}{169}$

3 a $\frac{2}{3}$ **b** $\frac{1}{2}$

c



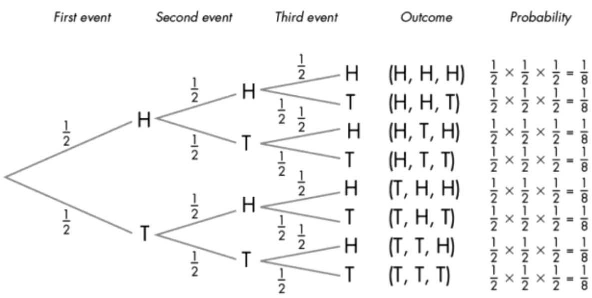
d i $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{5}{6}$

e 15 days

4 a $\frac{2}{5}$

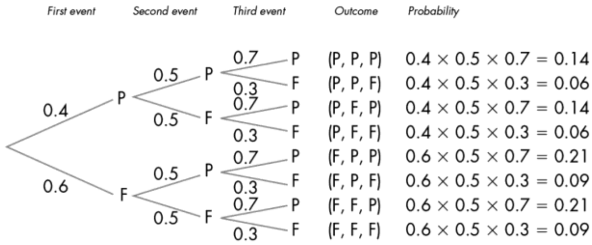
b i $\frac{4}{25}$ **ii** $\frac{12}{25}$

5 a



b i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

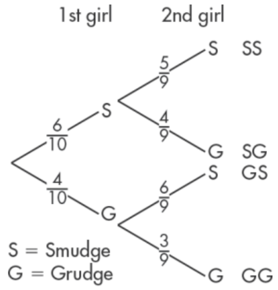
6 a



b 0.14 **c** 0.41 **d** 0.09

7 a $\frac{3}{5}$

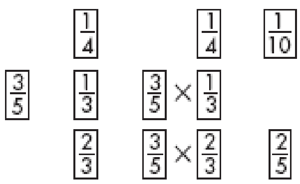
b



c i $\frac{1}{3}$ **ii** $\frac{7}{15}$ **iii** $\frac{8}{15}$

8 a 1 **b** 1

c



9 0.036

10 It will help to show all the 27 different possible events and which ones give the three different coloured sweets, then the branches will help you to work out the chance of each.

11 a $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ **b** $\left(\frac{1}{2}\right)^n$

Exercise 19D

1 a $\frac{4}{9}$ **b** $\frac{4}{9}$

2 a $\frac{1}{169}$ **b** $\frac{2}{169}$

3 a 0.08 **b** 0.32 **c** 0.48

4 $\left(\frac{1}{6}\right)^5 \times 6 = 0.000\ 77$

5 a $\frac{4}{25}$ **b** $\frac{9}{25}$ **c** $\frac{16}{25}$

6 a $\frac{3}{8}$ **b** $\frac{1}{120}$ **c** $\frac{119}{120}$

7 **a** **i** $\frac{1}{216} = 0.005$ **ii** $\frac{125}{216} = 0.579$

iii $\frac{91}{216} = 0.421$

b i $(\frac{1}{6})^n$ **ii** $(\frac{5}{6})^n$ **iii** $1 - (\frac{5}{6})^n$

8 **a** 0.54 **b** 0.216

9 **a** 0.343 **b** Independent events
c P(exactly two of the three cars are foreign) = P(FFB) + P(FBF) + P(BFF) = 3 × 0.7 × 0.7 × 0.3 = 0.441

10 $10 \times 0.6^9 \times 0.4 + 0.6^{10} = 0.046$

11 0.8

12 The events are not independent as he may already have a 10 or Jack or Queen or King in his hand, in which case the probability fraction will have a different numerator.

Exercise 19E

1 **a** $\frac{7}{10}$ **b** $\frac{2}{3}$ **c** $\frac{3}{8}$

2 **a** **i** $\frac{3}{8}$ **ii** $\frac{5}{8}$

b **i** $\frac{5}{12}$ **ii** $\frac{7}{12}$

c **i** $\frac{3}{20}$ **ii** $\frac{7}{20}$ **iii** $\frac{1}{2}$

3 **a** **i** $\frac{5}{9}$ **ii** $\frac{4}{9}$

b **i** $\frac{2}{3}$ **ii** $\frac{1}{3}$

c **i** $\frac{1}{3}$ **ii** $\frac{2}{15}$ **iii** $\frac{8}{15}$

4 **a** $\frac{1}{6}$ **b** 0

c **i** $\frac{2}{3}$ **ii** $\frac{1}{3}$ **iii** 0

5 **a** **i** $\frac{1}{120}$ **ii** $\frac{7}{40}$ **iii** $\frac{21}{40}$ **iv** $\frac{7}{24}$
b They are mutually exclusive and exhaustive events

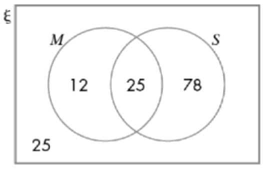
6 Both events are independent, the probability of seeing a British made car is always ¼

7 **a** 0.54 **b** 0.38 **c** 0.08
d They should add up to 1

8 First work out P(first blue) and P(second blue) remembering that the numerator and the denominator will each be one less than for P(first blue). Now work out P(first blue) × P(second blue). Then work out P(first white) and P(second white) remembering that the numerator and the denominator will each be one less than for P(first white). Now work out P(first white) × P(second white). Finally add together the two probabilities.

9 $\frac{1}{270\,725}$

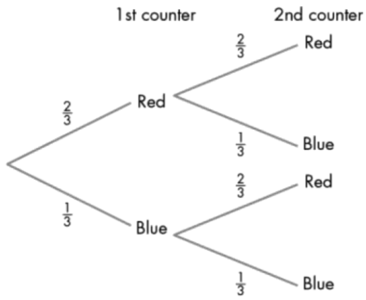
10 **a**



b **i** $\frac{12}{140} = 0.086$ **ii** $\frac{25}{103} = 0.243$

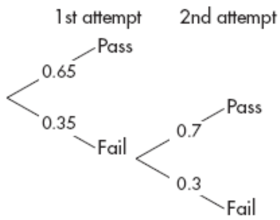
Review questions

1 **a**



b $\frac{5}{9}$ **c** $\frac{4}{9}$

2 **a**



b **i** 0.895 **ii** 0.105
c Calculate 0.895^2

3 $\frac{4}{15}$

4 $\frac{11}{30}$

5 Work out $\frac{5}{16} \div \frac{3}{8}$ $(= \frac{5}{6})$

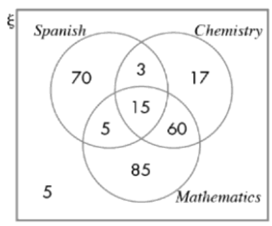
6 0.045

7 0.375

8 **a** $\frac{13}{20}$ as it cannot be square rooted

b $\frac{1}{9}$ as this gives a ratio of red to blue of 1 : 2

9 **a**



b **i** $\frac{70}{260} = 0.269$ **ii** $\frac{60}{260} = 0.231$
iii $\frac{5}{260} = 0.0192$ **iv** $\frac{20}{165} = 0.121$

Chapter 20 – Geometry and Measures: Properties of circles

Exercise 20A

Students' own work.

Exercise 20B

1 **a** 56° **b** 62° **c** 105° **d** 55° **e** 45°
f 30° **g** 60° **h** 145°

2 **a** 55° **b** 52° **c** 50° **d** 24° **e** 39°
f 80° **g** 34° **h** 30°

3 **a** 41° **b** 49° **c** 41°

4 **a** 72° **b** 37° **c** 72°

5 ∠AZY = 35° (angles in a triangle), *a* = 55° (angle in a semicircle = 90°)

6 **a** $x = y = 40°$ **b** $x = 131°$, $y = 111°$
c $x = 134°$, $y = 23°$ **d** $x = 32°$, $y = 19°$
e $x = 59°$, $y = 121°$ **f** $x = 155°$, $y = 12.5°$

7 ∠BED = 15° (angles at circumference from chord BD are equal); ∠EBC = 180° – (15° + 38°) = 127° (angles in a triangle): ∠ADC = 180° – (15° + 38°) = 127° (angles in a triangle); x = its vertically opposite angle which equals 360° – (127° + 127° + 38°) = 68° (angles in a quadrilateral). So Lana is correct, but Lex probably misread his calculator answer.

8 ∠ABC = 180° – x (angles on a line), ∠AOC = 360° – 2x (angle at centre is twice angle at circumference), reflex ∠AOC = 360° – (360° – 2x) = 2x (angles at a point)

9 **a** x **b** 2x
c Circle theorem 1 states from an arc AB, any point subtended from the arc on the circumference is half the angle subtended at the centre. So where this arc AB is the diameter, the angle subtended at the centre is 1 straight line and so 180°, the angle at the circumference then is half of 180 which is 90°, a right angle.

10 20°

11 It follows from theorem 1 that wherever point C is on the circumference, the angle subtended from arc AB at the circumference is always half the angle subtended at the centre. So every possible angle subtended at the circumference from arc AB will have the same angle at the centre and hence the same angle at the circumference. This proves circle theorem 3.

Exercise 20C

1 **a** $a = 50°$, $b = 95°$
b $c = 92°$, $x = 90°$
c $d = 110°$, $e = 110°$, $f = 70°$
d $g = 105°$, $h = 99°$
e $j = 89°$, $k = 89°$, $l = 91°$
f $m = 120°$, $n = 40°$
g $p = 44°$, $q = 68°$
h $x = 40°$, $y = 34°$

2 **a** $x = 26°$, $y = 128°$ **b** $x = 48°$, $y = 78°$
c $x = 133°$, $y = 47°$ **d** $x = 36°$, $y = 72°$
e $x = 55°$, $y = 125°$ **f** $x = 35°$
g $x = 48°$, $y = 45°$ **h** $x = 66°$, $y = 52°$

3 **a** Each angle is 90° and so opposite angles add up to 180° and hence a cyclic quadrilateral.
b One pair of opposite angles are obtuse, i.e. more than 90°, hence their sum will be more than 180°.

4 **a** $x = 49°$, $y = 49°$ **b** $x = 70°$, $y = 20°$
c $x = 80°$, $y = 100°$ **d** $x = 100°$, $y = 75°$

5 **a** $x = 50°$, $y = 62°$ **b** $x = 92°$, $y = 88°$
c $x = 93°$, $y = 42°$ **d** $x = 55°$, $y = 75°$

6 **a** $x = 95°$, $y = 138°$ **b** $x = 14°$, $y = 62°$
c $x = 32°$, $y = 48°$ **d** $x = 52°$

7 **a** 54.5° **b** 125.5° **c** 54.5°

8 **a** $x + 2x – 30° = 180°$ (opposite angles in a cyclic quadrilateral), so $3x – 30° = 180°$
b $x = 70°$, so $2x – 30° = 110°$ ∠DOB = 140° (angle at centre equals twice angle at circumference), $y = 80°$ (angles in a quadrilateral)

9 **a** x
b 360° – 2x
c,d ∠ADC = $\frac{1}{2}$ reflex ∠AOC = 180° – x, so ∠ADC + ∠ABC = 180°

10 Let ∠AED = x, then ∠ABC = x (opposite angles are equal in a parallelogram), ∠ADC = 180° – x (opposite angles in a cyclic quadrilateral), so ∠ADE = x (angles on a line)

11 18°

Exercise 20D

1 **a** 38° **b** 110° **c** 15° **d** 45°

2 **a** 6 cm **b** 10.8 cm **c** 3.21 cm **d** 8 cm

3 **a** $x = 12°$, $y = 156°$ **b** $x = 100°$, $y = 50°$
c $x = 62°$, $y = 28°$ **d** $x = 30°$, $y = 60°$

4 **a** 62° **b** 66° **c** 19° **d** 20°

5 19.5 cm

6 ∠OCD = 58° (triangle OCD is isosceles), ∠OCB = 90° (tangent/radius theorem), so ∠DCB = 32°, hence triangle BCD is isosceles (2 equal angles)

7 a $\angle AOB = \cos^{-1}\frac{OA}{OB} = \cos^{-1}\frac{OC}{OB} = \angle COB$

b As ∠AOB = ∠COB, so ∠ABO = ∠CBO, so OB bisects ∠ABC

8 If the tangent XY touches the circle at C, then CY = 10 cm. ∠ OYC = 30° (theorem 7). Hence where *r* is the radius of the circle, then *r*/10 = tan 30°, hence *r* = 10 × tan 30° = 5.7735, So Ling is correct to one decimal place.

9 38°

Exercise 20E

Students' own work.

Exercise 20F

1 a a = 65°, b = 75°, c = 40°
b d = 79°, e = 58°, f = 43°
c g = 41°, h = 76°, i = 76°
d k = 80°, m = 52°, n = 80°

2 a a = 75°, b = 75°, c = 75°, d = 30°
b a = 47°, b = 86°, c = 86°, d = 47°
c a = 53°, b = 53°
d a = 55°

3 a 36° **b** 70°

4 a x = 25°
b x = 46°, y = 69°, z = 65°
c x = 38°, y = 70°, z = 20°
d x = 48°, y = 42°

5 ∠ACB = 64° (angle in alternate segment), ∠ACX = 116° (angles on a line), ∠CAX = 32° (angles in a triangle), so triangle ACX is isosceles (two equal angles)

6 ∠AXY = 69° (tangents equal and so triangle AXY is isosceles), ∠XZY = 69° (alternate segment), ∠XYZ = 55° (angles in a triangle)

7 a $2x$ **b** $90° – x$
c OPT = 90°, so APT = x

8 Mark any point R on the circumference of the circle so that angle QRT is in the alternate segment to angle PTQ. Draw triangle QRT.
Then:
Angle PTQ = angle QRT (alternate segment theorem)
Angle PQT = angle QRT (alternate segment theorem)
Therefore angle PTQ = angle PQT and triangle PQT is isosceles.
Hence PQ = PT.
Other proofs may be possible.

Review questions

1 a 44°, both angles subtended from the same chord
b 52°, each vertex touches the circumference
c 140°, the three points not the centre are touching the circumference

2 a 55° **b** 75°

3 a DOB is double DAB
b DAB and DCB add up to 180° since ABCD is a cyclic quadrilateral

4 To be a rhombus, DOB must equal DCB, you know that DOB = $2x$ (double DAB), you know that DCB = 180° – x (as ABCD is a cyclic quadrilateral), so $2x$ = 180° – x, hence $3x$ = 180° → x = 60°

5 TPR = 42°, alternate segment; PRQ = 42°, alternate angles in parallel lines; RPQ = 42°, isosceles triangle; PQR = 180° – 2 × 42° = 96°, angles in a triangle; PTR = 180° – 96° = 84°, opposite angles in a cyclic quadrilateral.

6 CAO = 90° – 66° = 24°; ACO = 24°, isosceles triangle; AOB = 360° – (2 × 90° + 50°) = 130°, angles in quadrilateral; ACB = 130° ÷ 2 = 65°, angles at centre double angle at circumference; OCB = 65° – 24° = 41°; x = OCB, isosceles triangle hence x = 41°.

7 OCA = 90° and OBA = 90° as AB and AC are both tangents to the circle, centre O. This is a pair of opposite angles having the sum of 180°. Since the sum of the angles is 360°, the other pair of angles BC and BOC will add up to 360° – 180° = 180°, so both pairs of opposite angles sum to 180°, hence it is a cyclic quadrilateral.

8 OBA = 90° – x; OAB = 90° – x, angles in an isosceles triangle. BOA = 180° – (90° – x + 90° – x) = 180° – (180° – $2x$) = 180° – 180° + $2x$ = $2x$

9 Using Pythagoras' theorem, OC = $\sqrt{8^2 + 12^2}$ = 14.4 (3sf); PC = 14.4 – 8 = 6.4 (2 sf); the answer is given to 2 sf with the assumption that the 12 is 2 sf.

Chapter 21 – Ratio, proportion and rates of change: Variation

Exercise 21A

1 a 15 **b** 2

2 a 75 **b** 6

3 a 150 **b** 6

4 a 22.5 **b** 12

5 a 175 miles **b** 8 hours

6 a £66.50 **b** 175 kg

7 a 44 **b** 84 m^2

8 a 50 **b** Spaces = $\frac{1}{14}$ area

9 17 minutes 30 seconds

10 22.5 cm

Exercise 21B

1 a 100 **b** 10

2 a 27 **b** 5

3 **a** 56 **b** 1.69

4 **a** 192 **b** 2.25

5 **a** 25.6 **b** 5

6 **a** 80 **b** 8

7 **a** £50 **b** 225

8 **a** 3.2 °C **b** 10 atm

9 **a** 388.8 g **b** 3 mm

10 **a** 2 J **b** 40 m/s

11 **a** £78 **b** 400 miles

12 4000 cm^3

13 £250

14 **a** B **b** A **c** C

15 **a** B **b** A

16 $S = kM^{\frac{2}{3}}$

Exercise 21C

1 $Tm = 12$ **a** 3 **b** 2.5

2 $Wx = 60$ **a** 20 **b** 6

3 $Q(5 - t) = 16$ **a** –3.2 **b** 4

4 $Mt^2 = 36$ **a** 4 **b** 5

5 $W\sqrt{T} = 24$ **a** 4.8 **b** 100

6 $x^3y = 32$ **a** 32 **b** 4

7 $gp = 1800$ **a** £15 **b** 36

8 $tD = 24$ **a** 3 °C **b** 12 km

9 $ds^2 = 432$ **a** 1.92 km **b** 8 m/s

10 $p\sqrt{h} = 7.2$ **a** 2.4 atm **b** 100 m

11 $W\sqrt{F} = 0.5$ **a** 5 t/h **b** 0.58 t/h

12 B – This is inverse proportion, as x increases y decreases.

13

x	8	27	64
y	1	$\frac{2}{3}$	$\frac{1}{2}$

14 4.3 miles

15 $F = 2.02 \times 10^{19}$ N

Review questions

1

x	25	100	400
y	10	20	40

2 **a** $E = 4000v$ **b** 3.6 m/s

3 **a** $y = 4x^{-\frac{1}{3}}$ or $y = \frac{4}{\sqrt[3]{x}}$

b **i** 20 **ii** 8

4 19.4 cm

5 128

6 **a** $D = 5M^2$ **b** 245 **c** 3

7 80

8 **a** 2.5 **b** 0.25 **c** 250 **d** 50, –50

9 **a** 10 **b** 3.375

10 **a** 48π **b** 9

11 **a** $A = \frac{100}{B^2}$ or $AB^2 = 100$ **b** 4

12 125

13 **a** 27 hertz **b** Cannot divide by 0

14 $a = 9$, $b = 144$

15 40

Chapter 22 – Geometry and measures: Triangles

Exercise 22A

1 13.1 cm

2 73.7°

3 9.81 cm

4 33.5 m

5 **a** 10.0 cm **b** 11.5° **c** 4.69 cm

6 PS = 4 tan 25 = 1.865 230 6, angle QRP = $\tan^{-1} \frac{7.8652306}{4}$ = 63.0, angle QRP = 63.0 – 25 = 38.0

7 774 m

8 **a** $\sqrt{2}$ cm

b **i** $\frac{\sqrt{2}}{2}$ (an answer of $\frac{1}{\sqrt{2}}$ would also be accepted)

ii $\frac{\sqrt{2}}{2}$ **iii** 1

9 The calculated answer is 14.057 869, so Eve is correct to give 14° as her answer. She could also have been correct to round off to 14.1°

Exercise 22B

1 25.1°

2 **a** 24.0° **b** 48.0° **c** 13.5 cm **d** 16.6°

3 **a** 58.6° **b** 20.5 cm **c** 2049 cm^3 **d** 64.0°

4 **a** 73.2° **b** £1508.31

5 **a** 3.46 m **b** 70.5°

6 For example, the length of the diagonal of the base is $\sqrt{b^2 + c^2}$ and taking this as the base of the triangle with the height of the edge, then the hypotenuse is
$\sqrt{(a^2 + (\sqrt{b^2 + c^2})^2)} = \sqrt{a^2 + b^2 + c^2}$

7 It is 44.6°; use triangle XDM where M is the midpoint of BD; triangle DXB is isosceles, as X is over the point where the diagonals of the base cross; the length of DB is $\sqrt{656}$, the cosine of the required angle is 0.5 $\sqrt{656} \div 18$.

Exercise 22C

1a

x	sin *x*	*x*	sin *x*	*x*	sin *x*	*x*	sin *x*
0°	0	180°	0	180°	0	360°	0
15°	0.259	165°	0.259	195°	–0.259	345°	–0.259
30°	0.5	150°	0.5	210°	–0.5	330°	–0.5
45°	0.707	135°	0.707	225°	–0.707	315°	–0.707
60°	0.866	120°	0.866	240°	–0.866	300°	–0.866
75°	0.966	105°	0.966	255°	–0.966	285°	–0.966
90°	1	90°	1	270°	–1	270°	–1

b They are the same for values between 90° and 180°. They have the opposite sign for values between 180° and 360°

2 **a** Sine graph
b Line symmetry about x = 90, 270 and rotational symmetry about (180, 0)

Exercise 22D

1a

x	cos x	x	cos x	x	cos x	x	cos x
0°	1	180°	–1	180°	–1	360°	1
15°	0.966	165°	–0.966	195°	–0.966	345°	0.966
30°	0.866	150°	–0.866	210°	–0.866	330°	0.866
45°	0.707	135°	–0.707	225°	–0.707	315°	0.707
60°	0.5	120°	–0.5	240°	–0.5	300°	0.5
75°	0.259	105°	–0.259	255°	–0.259	285°	0.259
90°	0	90°	0	270°	0	270°	0

b Negative cosines are between 90 and 270, the rest are positive,

2 **a** Cosine graph
b Line symmetry about x = 180°, rotational symmetry about (90°, 0) , (270°, 0)

Exercise 22E

1 **a** 36.9°, 143.1° **b** 53.1°, 126.9°
c 48.6°, 131.4° **d** 224.4°, 315.6°
e 194.5°, 345.5° **f** 198.7°, 341.3°
g 190.1°, 349.9° **h** 234.5°, 305.5°
i 28.1°, 151.9° **j** 185.6°, 354.4°
k 33.6°, 146.4° **l** 210°, 330°

2 sin 234°, as the others all have the same numerical value.

3 **a** 438° or 78° + 360n°
b –282° or 78° – 360n°
c Line symmetry about ±90n° where n is an odd integer.
Rotational symmetry about ±180n° where n is an integer.

4 30°, 150°, 199.5°, 340.5°

5 **a** 53.1°, 306.9° **b** 54.5°, 305.5°
c 62.7°, 297.3° **d** 54.9°, 305.1°
e 79.3°, 280.7° **f** 143.1°, 216.9°
g 104.5°, 255.5° **h** 100.1°, 259.9°
i 111.2°, 248.8° **j** 166.9°, 193.1°
k 78.7°, 281.3° **l** 44.4°, 315.6°

6 cos 58°, as the others are negative.

7 **a** 492° or 132° + 360n°
b –228° or 132° – 360n°
c Line symmetry about ±180n° where n is an integer.
Rotational symmetry about ±90n° where n is an odd integer.

8 **a** **i** High tides 0940, 2200, low tides 0300, 1520
ii 12hrs 20min
b **i** Same periodic shape
ii The period of the cycle is in time not degrees, no negative values on the y axis

Exercise 22F

1 **a** 0.707 **b** –1 (–0.9998)
c –0.819 **d** 0.731

2 **a** –0.629 **b** –0.875
c –0.087 **d** 0.999

3 **a** 21.2°, 158.8° **b** 209.1°, 330.9°
c 50.1°, 309.9° **d** 150.0°, 210.0°
e 60.9°, 119.1° **f** 29.1°, 330.9°

4 30°, 150°

5 –0.755

6 **a** 1.41 **b** –1.37 **c** –0.0367 **d** –0.138
e 1.41 **f** –0.492

7 True

8 **a** cos 65° **b** cos 40°

9 **a** 10°, 130° **b** 12.7°, 59.3°

10 38.2°, 141.8°

11 $\sin^{-1}$ 0.9659 = 74.994 28, which is 75 to 2 sf.
435 = 75 + 360. From the sine curve extended, sine 75 = sine 435.
Rose is therefore correct as she has rounded her solution.
Keiren could also be correct as the answer could also be given more accurately as 434.9942838

Exercise 22G

1 **a** Maths error **b** 89.999 999

2 5 729 577 951

3 **a–b** Graph of tan x **c** All 0

d Students' own explanations

4 **a** Tan is positive for angles between 0–90° and 180–270°
b Yes, 180°

Exercise 22H

1 **a** 41.2°, 221.2° **b** 123.7° and 303.7°

2 **a** 14.5°, 194.5° **b** 61.9°, 241.9°
c 68.6°, 248.6° **d** 160.3°, 340.3°
e 147.6°, 327.6° **f** 105.2°, 285.2°
g 54.4°, 234.4° **h** 130.9°, 310.9°
i 174.4°, 354.4° **j** 44.9°, 224.9°

2 tan 235°, as the others have a numerical value of 1

3 **a** 425° or 65° + 180n°, $n > 2$
b –115° or 65° – 180n°
c No line symmetry
Rotational symmetry about ±180n° where n is an integer.

5 $\tan^{-1}$ 0.4040 = 21.9987 which is 22 to 2 sf, so $\tan^{-1}$ (–0.4040) is same as tan 180 – 22 = 158.
If you calculate $\tan^{-1}$(–0.4040) on your calculator it will give –21.9987 = –22 (2 sf).
Mel is therefore correct as he has rounded his solution. Jose is also correct.

Exercise 22I

1 **a** 3.64 m **b** 8.05 cm **c** 19.4 cm

2 **a** 46.6° **b** 112.0° **c** 36.2°

3 50.3°, 129.7°

4 This statement can be shown to be true by using $\frac{a}{\sin A} = \frac{b}{\sin B}$. As $a = b \times \frac{\sin A}{\sin B}$,
if $a > b$ then $\sin A > \sin B$ and so $\frac{\sin A}{\sin B} > 1$,
hence $b \times \frac{\sin A}{\sin B} > b$.

5 2.88 cm, 20.9 cm

6 **a** **i** 30° **ii** 40°
b 19.4 m

7 36.5 m

8 22.2 m

9 3.47 m

10 The correct height is 767 m. Paul has mixed the digits up and placed them in the wrong order.

11 26.8 km/h

12 64.6 km

13 Check students' answers.

14 134°

15 Check that proof is valid.

Exercise 22J

1 **a** 7.71 m **b** 29.1 cm **c** 27.4 cm

2 **a** 76.2° **b** 125.1° **c** 90°
d Right-angled triangle

3 5.16 cm

4 65.5 cm

5 **a** 10.7 cm **b** 41.7° **c** 38.3°
d 6.69 cm
e 54.4 cm^2

6 72.3°

7 25.4 cm, 38.6 cm

8 58.4 km at 092.5°

9 21.8°

10 **a** 82.8° **b** 8.89 cm

11 42.5 km

12 Check students' answers.

13 111°; the largest angle is opposite the longest side

Exercise 22K

1 **a** 8.60 m **b** 90° **c** 27.2 cm
d 26.9° **e** 41.0° **f** 62.4 cm

2 7 cm

3 11.1 km

4 19.9 knots

5 **a** 27.8 miles
b 262°

6 **a** A = 90°; this is Pythagoras' theorem
b A is acute
c A is obtuse

7 The answer is correct to 3 sf but the answer could be slightly less accurate (as 140 m to 2 sf) since the question data is given to 2 sf

Exercise 22L

1 **a** 24.0 cm^2 **b** 26.7 cm^2 **c** 243 cm^2
d 21 097 cm^2 **e** 1224 cm^2

2 4.26 cm

3 **a** 42.3° **b** 49.6°

4 103 cm^2

5 2033 cm^2

6 21.0 cm^2

7 **a** 33.2° **b** 25.3 cm^2

8 Check that proof is valid.

9 21 cm^2

10 726 cm^2

11 $\frac{a^2\sqrt{3}}{4}$

12 c

Review questions

1 cos A = $\frac{12^2+10^2-15^2}{2\times12\times10}$ = 0.079 166, $\cos^{-1}$ 0.079 166 = 85.459 371 = 85.5° (3 sf), so Oliver is incorrect, he has truncated the final answer to 3 figures instead of rounding off.

2 area = $\frac{1}{2}$ × 7 × 13 × sin 116 = 40.895129 = 40.9 (3 sf)

3 $AB^2 = 10^2 + 11^2 - 2 \times 10 \times 11 \times \cos 70$ = 145.755 57, AB = 12.1 (3 sf). The longest side of a triangle is opposite the largest angle, so as AB is the longest side, then angle C must be the largest angle

4 **a** **i** Let QP = 1, then QT = $\frac{1}{2}$, angle QPT = 30°, sin 30 = $\frac{\frac{1}{2}}{1}$ = $\frac{1}{2}$

ii $PT^2 = 1^2 - (\frac{1}{2})^2 = \frac{3}{4}$, so PT = $-\frac{\sqrt{3}}{2}$, hence cos 30 = $\frac{\frac{\sqrt{3}}{2}}{1}$ = $\frac{\sqrt{3}}{2}$

b $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$

c Assume QPT is any right-angled triangle with angle PQT as θ and PQ = 1. Then QT = cos θ, PT = sin θ, so using Pythagoras, where $QT^2 + PT^2 = PQ^2$, $(\sin x)^2 + (\cos x)^2 = 1$

5 22.2 m

6 60°, 109.5°, 250.5°, 300°

7 58.8°

8 FB = $\sqrt{5^2+6^2} = 7.81$ cm

AF = $\sqrt{9^2+7.81^2} = 11.92$ cm

So ∠AFD = $\sin^{-1}(\frac{6}{11.92}) = 30.2°$

9 Jamil is correct to 1 dp

Chapter 23 – Algebra: Graphs

Exercise 23A

1 **a** **i** 2 h **ii** 3 h **iii** 5 h
b **i** 40 km/h **ii** 120 km/h **iii** 40 km/h

2 **a** 10 mph
b Faster. The graph is steeper.
c $13\frac{1}{3}$ mph

3 **a** $2\frac{1}{2}$ km/h **b** 3.75 m/s **c** $2\frac{1}{2}$ km/h

4 **a** 30 km **b** 40 km **c** 100 km/h

5 **a** **i** 263 m/min (3 sf) **ii** 15.8 km/h (3 sf)
b **i** 500 m/min
c Paul by 1 minute

6 **a** Patrick ran quickly at first, then had a slow middle section but he won the race with a final sprint. Araf ran steadily all the way and came second. Sean set off the slowest, speeded up towards the end but still came third.
b **i** 1.67 m/s **ii** 6 km/h

7 There are three methods for doing this question. This table shows the first, which is writing down the distances covered each hour.

Time	9am	9:30	10:00	10:30	11:00	11:30	12:00	12:30
Walker	0	3	6	9	12	15	18	21
Cyclist	0	0	0	0	7.5	15	22.5	30

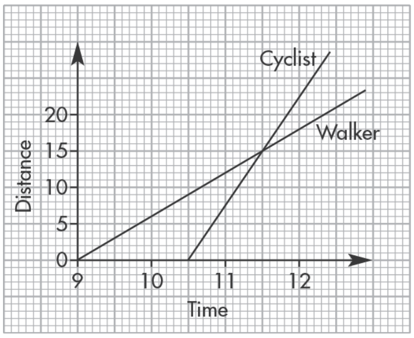
The second method is algebra:

Walker takes T hours until overtaken, so $T = \frac{D}{6}$;

Cyclist takes $T - 1.5$ to overtake, so $T - 1.5 = \frac{D}{15}$.

Rearranging gives $15T - 22.5 = 6T$, $9T = 22.5$, $T = 2.5$.

The third method is a graph:



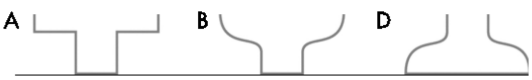
All methods give the same answer of 11:30 when the cyclist overtakes the walker.

8 **a** Vehicle 2 overtook Vehicle 1
b Vehicle 1 overtook Vehicle 2
c Vehicles travelling in different directions
d Vehicle 2 overtook Vehicle 1
e 17:15
f 32.0 mph if you only count travelling time, or 11.3 mph if you count total time.

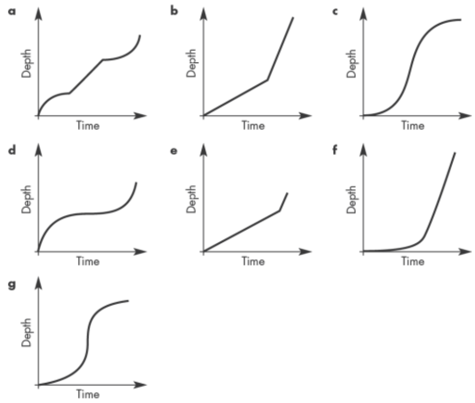
Exercise 23B

1 **a** Two taps on **b** One tap on
c Shejuti gets in the bath
d Shejuti has a bath
e Shejuti takes the plug out, water leaves the bath
f Shejuti gets out of the bath
g Water continues to leave the bath until the bath is empty

2 **a** Graph C
b



3



Exercise 23C

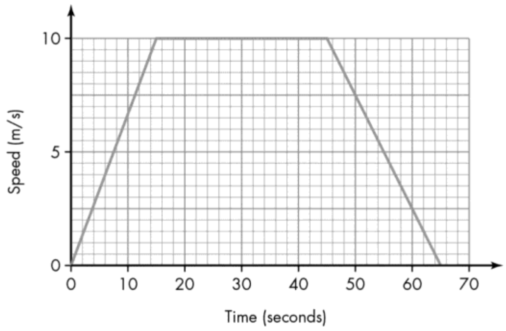
1 **a** 20 m/s **b** 100 metres
c 150 metres **d** 750 metres

2 **a** 15 km **b** 5 km/h

3 **a** AB, greatest area **b** 45 miles **c** 135 miles

4 15 m/s

5 **a**



b 475 metres

6 **a** Could be true or false
b Must be true
c Must be true
d Could be true or false
e Could be true or false

Exercise 23D

1 **a** 6 m/s^2 **b** 3 m/s^2 **c** 20 s
d 1200 m **e** 2100 m

2 40 km/h^2, 30 km/h^2, 100 km/h^2 **b** 52.5 km

3 **a** $\frac{3v}{10}$ m/s^2 **b** 337.5 m

Exercise 23E

1 **a** 80 miles, underestimate
b 250 metres, overestimate
c 75 metres, underestimate
d 180 metres, underestimate

2 **a** 8 miles, underestimate
b 40 miles, overestimate

3 **a** **i** 10 m/s **ii** 40 m/s
b **i** 900 m
ii Around 1070 m, depending on student's division of the shape
c **ii** is more accurate because the shapes are closer to the curve

4 **a** Car starts from rest and speeds up to 10 m/s after 20 seconds. It then travels at a constant speed of 10 m/s for 30 seconds, and then speeds up again to reach 20 m/s in the next 10 seconds.
b 120 metres

5 **a** The lorry increases speed at a steady rate whereas the car speeds up quickly at first but then levels off to a constant speed and then speeds up at an increasing rate to reach 20 m/s.
b Lorry travels further (600 metres as against car, approximately 550 metres) as area under graph is greater.

Exercise 23F

1 **a** Tangent drawn **b** 10 m/s **c** 0 m/s

2 **a** **i** 10–12 km/h **ii** 20–22 km/h
b 2 hours
c **i** 10 km/h **ii** 19 km/h

3 **a** About 1.8 m/s^2
b About 0.9 m/s^2
c About 1.8 m/s^2
d 20 s, gradient is zero because this is a maximum point

4 Any two points where the gradient of one is the negative of the other, e.g. at 1 s and 3 s.

Exercise 23G

1 **a** 6 **b** $2\sqrt{3}$ **c** $5\sqrt{3}$ **d** 24

2 **a** $6\sqrt{13}$ **b** 176 **c** 114 **d** $\frac{3}{2}$

3 **a** Inside **b** Outside **c** On circumference
d On circumference
e Outside **f** Inside

4 **a** Any three points such that $x^2 + y^2 = 25$
b 12

5 **a** $\frac{1}{2}$ **b** –2
c $y = -2x + 10$

6 Check proof is valid.

7 **a** $y = \frac{3}{5}x - \frac{34}{5}$ **b** $y = -\frac{1}{3}x - \frac{20}{3}$
c $y = -\frac{p}{q}x + \frac{p^2 + q^2}{q}$ (or $\frac{a^2}{q}$)

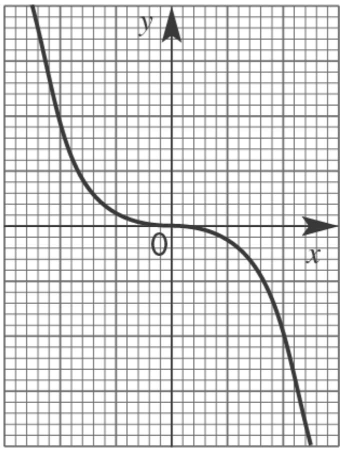
8 $y = 2x - 15$, $y = 2x + 15$

9 **b** $x + y = 10$, $x + y = -10$

10 **a** 10
b $x^2 + y^2 = 90$

Exercise 23H

1



2 **a** Values of *y*: –54, –31.25, –16, –6.75, –2, –0.25, 0, 0.25, 2, 6.75, 16, 31.25, 54

b 39.4

3 **a** Values of *y*: –24, –12.63, –5, –0.38, 2, 2.9, 3, 3.13, 4, 6.38, 11, 18.63, 30

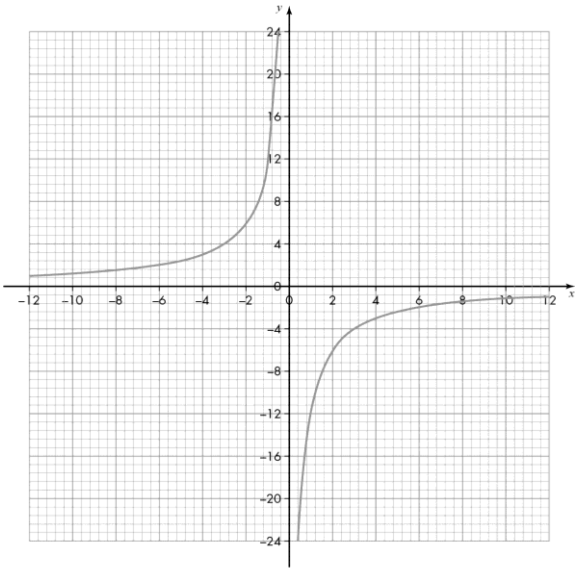
b 4.7 **c** –1.4 to –1.5

4 **a** Values of *y*: –16, –5.63, 1, 4.63, 6, 5.88, 5, 4.13, 4, 5.38, 9, 15.63, 26

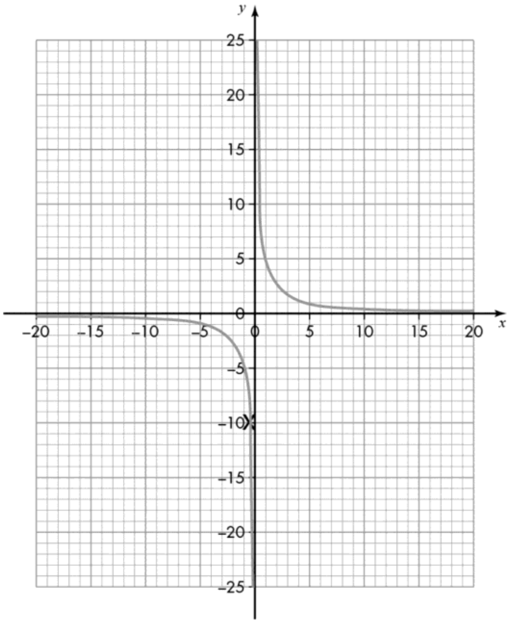
b **i** –2.1 **ii** (–0.8, 6)

iii (0.7, 3.9) **iv** (0, 5)

5 Values of *y*: 1, 2, 3, 4, 6, 12, 24, –24, –12, –6, –4, –3, –2, –1

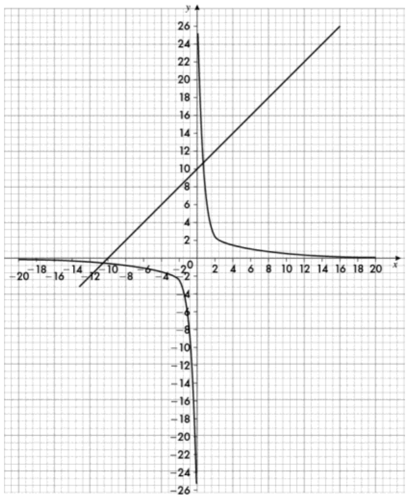


6 **a** Values of *y*: –0.25, –0.33, –0.5, –1, –2.5, –5, –10, –12.5, –25, 25, 12.5, 10, 5, 2.5, 1, 0.5, 0.33, 0.25



b Can't divide by 0

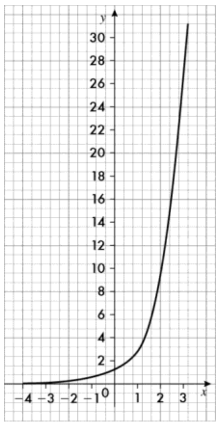
c



d 0.48 and −10.48

7 **a** Values of *y*: 0.01, 0.04, 0.11, 0.33, 1, 3, 9, 27

b 15.6 **c** –0.63



8 **a** Quadratic **b** Linear **c** Exponential

d Reciprocal **e** None **f** Cubic

g Linear **h** None **i** Quadratic

9 a The numbers go 1, 2, 4, … which is equivalent to 2^0, 2^1, 2^2, … so the formula is $2^{(n-1)}$

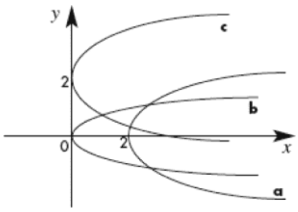
b $2^{63} = 9.22 \times 10^{18}$ **c** £4.61×10^{14}

10 a Number of pieces = 2^n so $2^{50} = 1.1 \times 10^{15}$ pieces

b 1.13×10^8 km

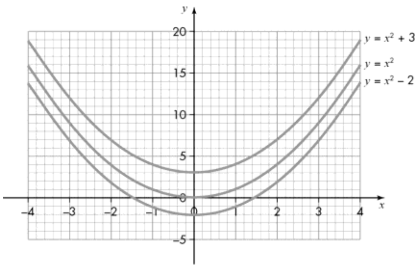
11 $a = 5$, $b = 3$

12



Exercise 23I

1 a, b



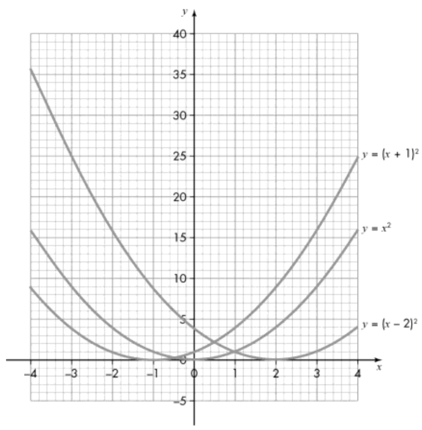
c $y = x^2 + 3$ is 3 units higher than $y = x^2$

d $y = x^2 - 2$ is 2 units lower than $y = x^2$

e i $y = x^2 + 6$ is 6 units higher than $y = x^2$

ii $y = x^2 - 6$ is 6 units lower than $y = x^2$

2 a, b



c $y = (x - 2)^2$ is 2 units to the right of $y = x^2$

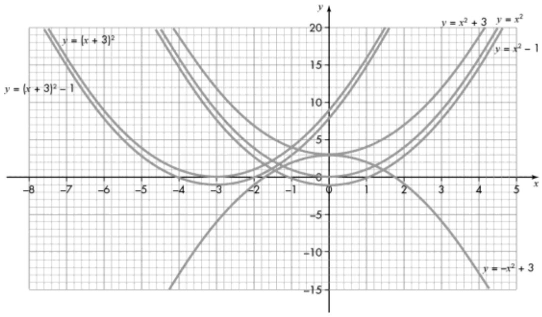
d $y = (x + 1)^2$ is 1 unit to the left of $y = x^2$

e i $y = (x - 3)^2$ is 3 units to the right of $y = x^2$

ii $y = (x + 4)^2$ is 4 units to the left of $y = x^2$

Exercise 23J

1 a

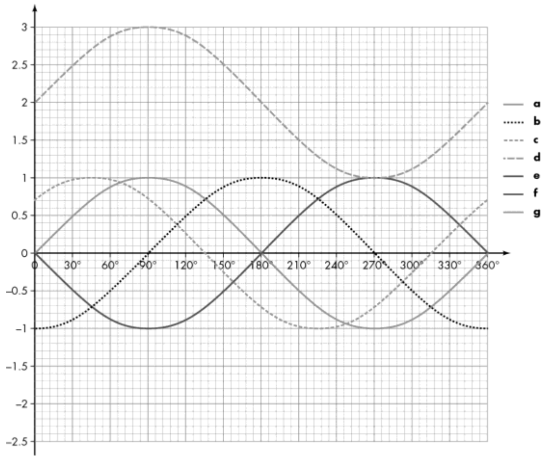


b Up 3 **c** Down 1 **d** 3 to the left

e 3 to the left and down 1

f Reflect in the x-axis and move up 3

2 a



b 90° to the left **c** 45° to the right

d Up 2 **e** Reflect in the x-axis

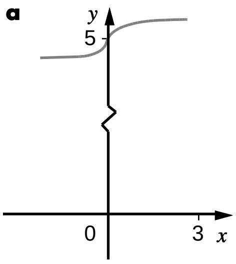
f Reflect in the y-axis

g Reflect in both axes

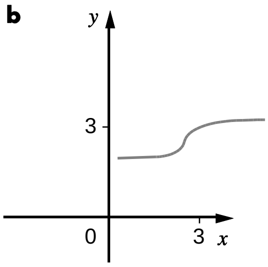
3 All of them.

4

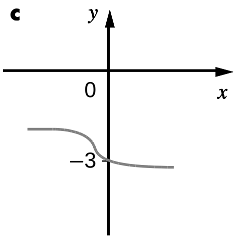
a



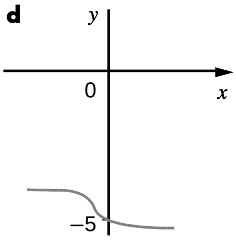
b



c

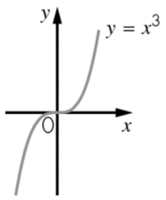


d

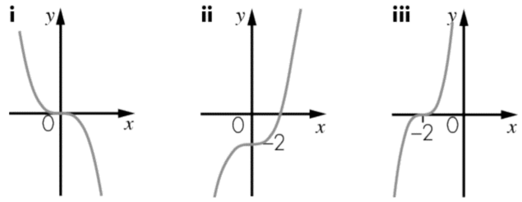


5 **a** $y = \cos x + 3$ **b** $y = \cos (x + 30°)$
c $y = \cos (x - 45°) - 2$

6 **a**



b



c **i** $y = -x^3$ **ii** $y = x^3 - 2$
iii $y = (x + 2)^3$

7 No, as $f(-x) = (-x)^2 = x^2$, and $-f(x) = -(x)^2 = -x^2$

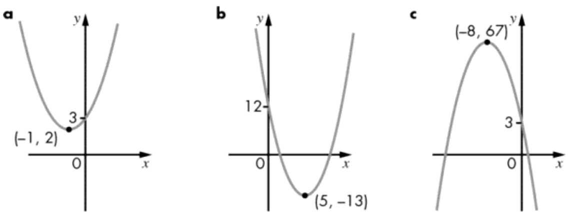
8 **a** $y = x^2 + 2$ **b** $y = (x - 2)^2$
c $y = -x^2 + 4$

9 **a** Translation
b **i** Equivalent **ii** Equivalent

Exercise 23K

1 **a** $y = f(x - 3) + 2$; 3 right and 2 up
b $y = f(x + 7) - 14$; 7 left and 14 down
c $y = f(x - 11) - 21$; 11 right and 21 down

2



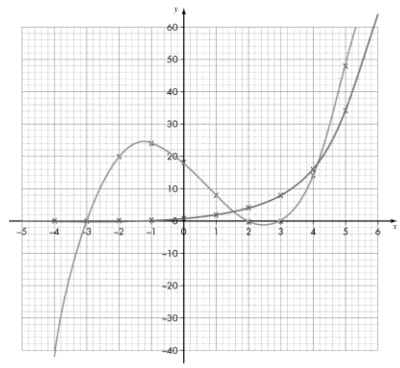
3 **a** $y = x^2 - 8x + 7$ **b** $y = -x^2 + 6x + 5$
c $y = x^2 - 14x + 59$

Review questions

1

2 **a** 19 **b** 0.7 m/s^2

3 **a**



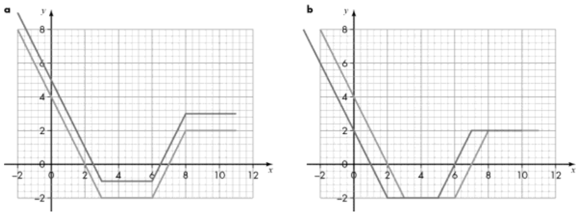
b –3.0, 1.6, 4.2

4 50

5 1F, 2C, 3D, 4A, 5B, 6E

6 **a** 5 **b** 4 **c** 320

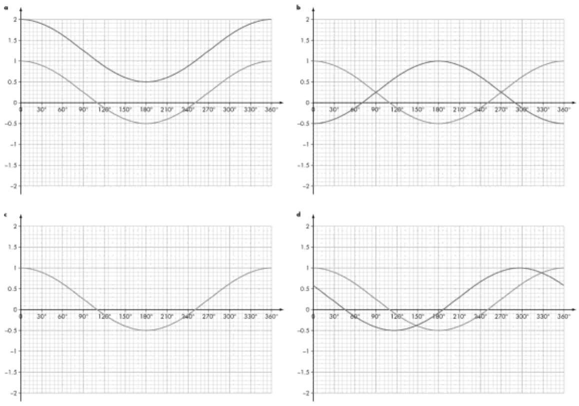
7



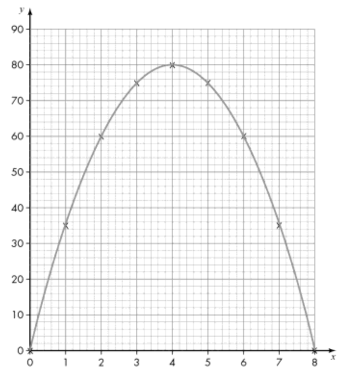
8 **a** (5, 5) and (7, 1)
b $\left(\frac{20}{3}, \frac{10}{3}\right)$

9 **a** –(0.8–0.9) m/h^2
b 8.6 miles

10



11 a



b 10 m/s

Chapter 24 – Algebra: Algebraic fractions and functions

Exercise 24A

1 **a** $\frac{5x}{6}$ **b** $\frac{23x}{20}$ **c** $\frac{x^2y+8}{4x}$

d $\frac{5x+7}{6}$ **e** $\frac{13x+5}{15}$ **f** $\frac{5x-10}{4}$

2 **a** $\frac{11x}{20}$ **b** $\frac{3x-2y}{6}$ **c** $\frac{xy^2-8}{4y}$

d $\frac{x+1}{4}$ **e** $\frac{x-1}{4}$ **f** $\frac{2-3x}{4}$

3 **a** $x = 3$ **b** $x = 2$ **c** $x = 0.75$ **d** $x = 3$

4 **a** $\frac{x^2}{6}$ **b** $\frac{8}{3}$ **c** $\frac{x^2-2x}{10}$

d $\frac{2x^2+x}{15}$ **e** $\frac{1}{2x}$

5 **a** $\frac{x}{y}$ **b** $\frac{2xy}{3}$ **c** $\frac{2x^2-12x+18}{75}$

d 1 **e** $\frac{1}{4x+2}$ **f** $\frac{x^2-5x+6}{48}$

g $\frac{1}{2x}$

6 **a** x **b** $\frac{x}{2}$ **c** $\frac{3x^2}{16}$ **d** 3

e $\frac{17x+1}{10}$ **f** $\frac{13x+9}{10}$ **g** $\frac{3x^2-5x-2}{10}$

h $\frac{x+3}{2}$ **i** $\frac{2}{3}$ **j** $\frac{2x^2-6y^2}{9}$

7 All parts: students' own working

8 **a** $\frac{x^2-8}{x^2-2}$ **b** 7

9 $\frac{x^2+14x+37}{x^3+12x^2+47x+60}$

10 $\frac{x+3}{x-6}$

11 First, he did not factorise and just cancelled the x^2s. Then he cancelled 2 and 6 with the wrong signs. Then he said two minuses make a plus when adding, which is not true.

12 $\frac{2x^2+x-3}{4x^2-9}$

13 **a** 3, –1.5 **b** 4, –1.25 **c** 3, –2.5 **d** 0, 1

14 **a** $\frac{x-1}{2x+1}$ **b** $\frac{2x+1}{x+3}$ **c** $\frac{2x-1}{3x-2}$

d $\frac{x+1}{x-1}$ **e** $\frac{2x+5}{4x-1}$

15 **a** Proof **b** 2 or $\frac{-10}{3}$

16 **a** Proof **b** 2 m/s

17 **a** $x^3+3\sqrt{2}\,x^2+6x+2\sqrt{2}$

b Proof **c** $99+70\sqrt{2}$

18 **a** $\frac{72-4x^2}{(x+3)^2(x-3)^2}$

b $\frac{4x^3+40x^2+122x+110}{(x+1)(x+2)(x+3)(x+4)}$

19 $\frac{x-\sqrt{3}}{x-3\sqrt{3}}$

Exercise 24B

1 **a** $c = \frac{p}{5}+3$ or $\frac{p+15}{5}$ **b** $c = \frac{15}{5-p}$

2 **a** $G = \frac{R}{F}-3$ or $\frac{R-3F}{F}$

b $G = \frac{R-3F}{F-1}$ or $\frac{3F-R}{1-F}$

3 **a** $a = \frac{b(q+p)}{q-p}$ **b** $b = \frac{a(q-p)}{q+p}$

c $a = \frac{b+5c}{4}$ **d** $r = \frac{A}{\pi(2h+k)}$

e $v = \frac{u}{\sqrt{(1-a)}}$ **f** $x = \frac{2R-3}{R-1}$

4 **a** $r = \frac{P}{\pi+2k}$ **b** $r = \sqrt{\frac{2A}{\pi+\sqrt{(k^2-1)}}}$

5 $P = \frac{100A}{100+RY}$

6 **a** $b = \frac{Ra}{a-R}$ **b** $a = \frac{Rb}{b-R}$

7 **a** $x = \frac{2+2y}{y-1}$

b $y - 1 = \frac{4}{x-2}$, $(x-2)(y-1) = 4$, $x - 2 = \frac{4}{y-1}$, $x = 2 + \frac{4}{y-1}$

c $y = 1 + \frac{4}{x-2} = \frac{x-2+4}{x-2} = \frac{x+2}{x-2}$ and $x = \frac{2+2y}{y-1}$

d Same formulae as in **a**

8 **a** Cannot take r as a common factor

b $\pi = \frac{3V}{r^2(2r+3h)}$

c Yes, $r = \sqrt[3]{\frac{3V}{5\pi}}$

9 $x = \frac{2W - 2zy}{z + y}$

10 $x = \frac{1-3y}{2y-5}$

The first number at the top of the answer is the constant term on the top of the original.
The coefficient of y at the top of the answer is the negative constant term on the bottom of the original.
The coefficient of y at the bottom of the answer is the coefficient of x on the bottom of the original.
The constant term on the bottom is negative the coefficient of x on the top of the original.

11 **a** Both are correct
b Alice's answer is easier to substitute into

Exercise 24C

1 **a i** 8 **ii** 14 **iii** 2 **iv** 4
b i 36 **ii** –9 **iii** 1241 **iv** –1.5

2 **a** 6 **b** $\frac{x+7}{4} - 3$ **c** 45

3 **a** 29 **b** 218 **c** 7.832

4 7

5 **a** 25 **b** 249 **c** 15 **d** 1807
e 1807 **f** 13 **g** ± 5

6 **a** 9 **b** –39 **c** –56 **d** –56
e 12 **f** 24.84 **g** ± 5

7 **a i** 54 **ii** 44 **b** 6 and –1

Exercise 24D

1 **a** $f^{-1}(x) = \frac{x+5}{4}$ **b** $f^{-1}(x) = \sqrt[3]{x-2}$

c $f^{-1}(x) = \frac{10}{x} - 1$ **d** $f^{-1}(x) = \frac{10-x}{2}$

e $f^{-1}(x) = 6x + 7$ **f** $f^{-1}(x) = \frac{3}{x-5}$

2 **a** $f^{-1}(x) = \frac{5x+2}{3x-1}$ **b** $-\frac{3}{2}$ **c** $f^{-1}(-\frac{3}{2}) = 1$

3 **a** Both inverse functions are the same as the original function.
b The inverse function is the same as the original function.
c Proof

Exercise 24E

1 **a** $8\frac{1}{2}$ **b** $6\frac{1}{2}$ **c** 43 **d** –2.25 **e** 5.8

2 **a** 48 **b** 229 **c** 18
d 29 **e** –8 **f** –141

3 **a i** $4x^3 - 32$ **ii** $11 - 4x$
iii $21 - 27x + 9x^2 - x^3$
iv $16x - 40$
v $x^9 - 18x^6 + 108x^3 - 222$
b $gh(x) = 4 - 4x$, $hg(x) = 11 - 4x$, $4 - 4x \neq 11 - 4x$

4 $\frac{1}{2}(b + 1)$

Exercise 24F

1 $x_2 = 4$ $x_3 = -10$ $x_4 = 88$ $x_5 = -598$

2 **a** 1878 **b** –4372 **c** –54.048 **d** 3

3 5.0701

4 $x_2 = 3.1414$ $x_3 = 3.1745$ $x_4 = 3.1821$
$x_5 = 3.1839$ $x_6 = 3.1843$

5 **a** 2.115 = 2.12 (2 dp)
b f(2.12) = 0.03 (2 dp)

6 Proof

7 **a** 3 and 7
b 7
c i Converges on 7
ii Diverges, towards square root of a negative
iii Converges on 7 **iv** stays on 3
d $x < 3$: diverges, towards square root of a negative
$x = 3$: stays on 3
$x > 7$: converges on 7

8 **a** Proof
b $x_2 = \frac{7}{2}$, $x_3 = 3$, $x_4 = \frac{7}{2}$
c $x_2 = \frac{29}{9}$, $x_3 = \frac{13}{4}$, $x_4 = \frac{29}{9}$
d $x_2 = \frac{73}{23}$, $x_3 = \frac{33}{10}$, $x_4 = \frac{73}{23}$
e $x_2 = 1 + \sqrt{5}$, $x_3 = 1 + \sqrt{5}$, $x_4 = 1 + \sqrt{5}$
f $x = 1 + \sqrt{5}$

9 **a** Proof **b** 67 cm^2
c This will depend upon how accurate the final value of x_{n+1} is

10 **a** 1 **b** 3

11 **a** Oscillates between 8.046, 0.148 and –2.262
b Diverges
c Converges on 2.707

Review questions

1 8

2 **a** $x = \frac{a + K}{C + 6}$ **b** 3.5

3 **a** $\frac{x + 3}{x}$ **b** $f^{-1}(x) = \frac{3}{x - 1}$

4 $\frac{7x}{3x + 1}$

5 **a** $x_1 = 2.54$, $x_2 = 2.57$, $x_3 = 2.58$, $x_4 = 2.59$
b 2.59 – it's the same

6 **a** $fg(x) = 3x^3 + 14$ **b** $3 \times 3^3 + 14 = 95$

7 **a** $f^{-1}(x) = \frac{x + q}{p}$ **b** $f^{-1}(x) = \sqrt[3]{a - x}$
c $f^{-1}(x) = \frac{a}{x} - c$

8 **a** **i** 2 **ii** 8 **iii** 18
iv 32 **v** 50
b $2n^2$

9 Proof

10 **a** $\frac{2x + 7}{x - 3}$ **b** $\frac{5}{6}$ or 16

11 $2 - 3xy = 4 - x$
mistake here expanding the brackets
$x = \frac{1 - 3y}{2}$ should be 2 divided by $(1 - 3y)$
corrected: $y = \frac{4 - x}{2 - 3x}$
$y(2 - 3x) = 4 - x$
$2y - 3xy = 4 - x$
$x - 3xy = 4 - 2y$
$x(1 - 3y) = 4 - 2y$
$x = \frac{4 - 2y}{1 - 3y}$
Hence $f^{-1}(x) = \frac{4 - 2x}{1 - 3x}$

12 **i** $\frac{3}{4}$ **ii** $-\frac{2}{3}$ **iii** 5 **iv** $\frac{8}{5}$

13 $-\frac{4}{x + 1}$

14 **a** 9.51, –10.5
b $x \le 10$, $x \ge 28$
c $-10 < x < 10$

15 **a** Proof **b** 5.31

16 **a** 21 **b** $a = -3$, $b = 2$

17 **a** Proof **b** 12

Chapter 25 – Geometry and measures: Vector geometry

Exercise 25A

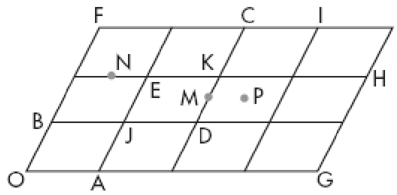
1 **a** Any three of: $\overrightarrow{AC}$, $\overrightarrow{CF}$, $\overrightarrow{BD}$, $\overrightarrow{DG}$, $\overrightarrow{GI}$, $\overrightarrow{EH}$, $\overrightarrow{HJ}$, $\overrightarrow{JK}$
b Any three of: $\overrightarrow{BE}$, $\overrightarrow{AD}$, $\overrightarrow{DH}$, $\overrightarrow{CG}$, $\overrightarrow{GJ}$, $\overrightarrow{FI}$, $\overrightarrow{IK}$
c Any three of: $\overrightarrow{AO}$, $\overrightarrow{CA}$, $\overrightarrow{FC}$, $\overrightarrow{IG}$, $\overrightarrow{GD}$, $\overrightarrow{DB}$, $\overrightarrow{KJ}$, $\overrightarrow{JH}$, $\overrightarrow{HE}$
d Any three of: $\overrightarrow{BO}$, $\overrightarrow{EB}$, $\overrightarrow{HD}$, $\overrightarrow{DA}$, $\overrightarrow{JG}$, $\overrightarrow{GC}$, $\overrightarrow{KI}$, $\overrightarrow{IF}$

2 **a** 2**a** **b** 2**b** **c** **a** + **b**
d 2**a** + **b** **e** 2**a** + 2**b** **f** **a** + 2**b**
g **a** + **b** **h** 2**a** + 2**b** **i** 3**a** + **b**
j 2**a** **k** **b** **l** 2**a** + **b**

3 **a** Equal **b** $\overrightarrow{AI}$, $\overrightarrow{BJ}$, $\overrightarrow{DK}$

4 **a** $\overrightarrow{OJ} = 2\overrightarrow{OD}$ and parallel
b $\overrightarrow{AK}$ **c** $\overrightarrow{OF}$, $\overrightarrow{BI}$, $\overrightarrow{EK}$

5



6 **a** Lie on same straight line
b All multiples of **a** + **b** and start at O
c H

7 **a** –**b** **b** 3**a** – **b** **c** 2**a** – **b**
d **a** – **b** **e** **a** + **b** **f** –**a** – **b**
g 2**a** – **b** **h** –**a** – 2**b** **i** **a** + 2**b**
j –**a** + **b** **k** 2**a** – 2**b** **l** **a** – 2**b**

8 **a** Equal but in opposite directions
b Any three of: $\overrightarrow{DA}$, $\overrightarrow{EF}$, $\overrightarrow{GJ}$, $\overrightarrow{FI}$, $\overrightarrow{AH}$

9 **a** Opposite direction and $\overrightarrow{AB} = -\frac{1}{2}\overrightarrow{CK}$
b $\overrightarrow{BJ}$, $\overrightarrow{CK}$
c $\overrightarrow{EB}$, $\overrightarrow{GO}$, $\overrightarrow{KH}$

10 506 mph on a bearing of 009°

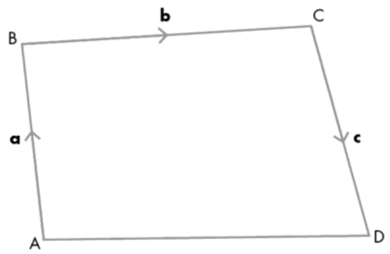
11 12 km/h on a bearing of 107°

12 **a** **i** **a** + **b** **ii** 3**a** + **b**
iii 2**a** – **b** **iv** 2**b** – 2**a**
b $\overrightarrow{DG}$ and $\overrightarrow{BC}$

13 Parts **b** and **d** could be, parts **a** and **c** could not be

14 **a** Any multiple (positive or negative) of 3**a** – **b**
b Will be a multiple of 3**a** – **b**

15 For example, let ABCD be a quadrilateral as shown.



Then $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$ = **a** + (**b** + **c**).

But $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD} = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$.
Hence $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$.

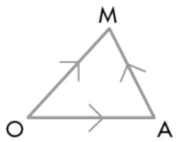
16 a i $2\mathbf{b} - 2\mathbf{a}$ **ii** $\mathbf{a} - \mathbf{c}$
iii $2\mathbf{c} - 2\mathbf{b}$ **iv** $\mathbf{b} + \mathbf{c} - \mathbf{a}$

b $\overrightarrow{RQ} = \mathbf{a} - \mathbf{c} = \overrightarrow{SP}$, similarly $\overrightarrow{PQ} = b = \overrightarrow{SR}$, so opposite sides are equal and parallel, hence PQRS is a parallelogram

Exercise 25B

1 a i $-\mathbf{a} + \mathbf{b}$ **ii** $\frac{1}{2}(-\mathbf{a} + \mathbf{b})$

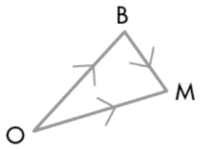
iii



iv $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

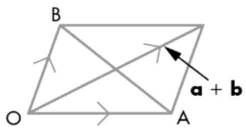
b i $\mathbf{a} - \mathbf{b}$ **ii** $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

iii



iv $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

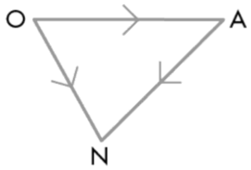
c



d M is midpoint of parallelogram of which OA and OB are two sides.

2 a i $-\mathbf{a} - \mathbf{b}$ **ii** $-\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

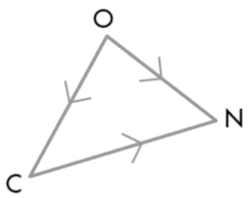
iii



iv $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

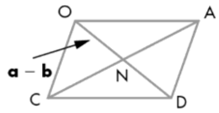
b i $\mathbf{b} + \mathbf{a}$ **ii** $\frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{a}$

iii



iv $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

c



d N is midpoint of parallelogram of which OA and OC are two sides

3 a i $-\mathbf{a} + \mathbf{b}$ **ii** $\frac{1}{3}(-\mathbf{a} + \mathbf{b})$ **iii** $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

b $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

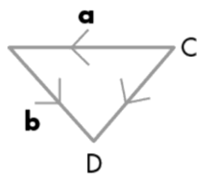
4 a i $\frac{2}{3}\mathbf{b}$ **ii** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ **iii** $-\frac{2}{3}\mathbf{b}$

b $\frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$

c $\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OE}$
$= \frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

d $\overrightarrow{DE}$ parallel to $\overrightarrow{CD}$ (multiple of $\overrightarrow{CD}$) and D is a common point

5 a



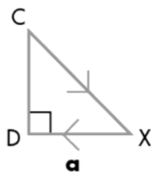
$\overrightarrow{CD} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

b i $-\mathbf{a}$ **ii** $-\mathbf{b}$ **iii** $\mathbf{a} - \mathbf{b}$

c 0, vectors return to starting point

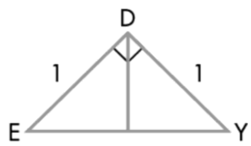
d i $2\mathbf{b}$ **ii** $2\mathbf{b} - 2\mathbf{a}$ **iii** $-2\mathbf{a}$
iv $2\mathbf{b} - \mathbf{a}$ **v** $-\mathbf{a} - \mathbf{b}$

6 a



$\overrightarrow{CX} = \sqrt{1^2 + 1^2}\,\mathbf{b} = \sqrt{2}\,\mathbf{b}$
$\overrightarrow{CD} = \overrightarrow{CX} + \overrightarrow{XD} = \sqrt{2}\,\mathbf{b} - \mathbf{a}$

b



$\overrightarrow{YE} = \sqrt{1^2 + 1^2}\,\mathbf{a} = \sqrt{2}\,\mathbf{a}$
$\overrightarrow{DE} = \overrightarrow{DY} + \overrightarrow{YE} = \mathbf{b} - \sqrt{2}\,\mathbf{a}$

c i $-\mathbf{a}$ **ii** $-\mathbf{b}$
iii $\mathbf{a} - \sqrt{2}\,\mathbf{b}$ **iv** $\sqrt{2}\,\mathbf{a} - \mathbf{b}$
v $\sqrt{2}\,\mathbf{a} + \mathbf{a}$ **vi** $\sqrt{2}\,\mathbf{b} + \mathbf{b}$
vii $2\mathbf{b} + \sqrt{2}\,\mathbf{b} - \mathbf{a} - \sqrt{2}\,\mathbf{a}$
viii $2\mathbf{b} + \sqrt{2}\,\mathbf{b} - 2\mathbf{a} - \sqrt{2}\,\mathbf{a}$

7 **a i** $-\mathbf{a} + \mathbf{b}$ **ii** $\frac{1}{2}(-\mathbf{a} + \mathbf{b}) = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

iii $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b i $\frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$ **ii** $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$

c i $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$ **ii** Equal

iii Parallelogram

d $\overrightarrow{AC} = -\mathbf{a} + \mathbf{c} = 2(-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}) = 2\,\overrightarrow{QM}$

8 **a i** $\frac{1}{2}\mathbf{a}$ **ii** $\mathbf{c} - \mathbf{a}$

iii $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$ **iv** $\frac{1}{2}\mathbf{c}$

b i $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ **ii** $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

c Opposite sides are equal and parallel

d NMRQ and PNLR

9 **a** $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b i Rhombus

ii They lie on a straight line, $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC}$

10 $k = 8$

11 **a** $\overrightarrow{YW} = \overrightarrow{YZ} + \overrightarrow{ZW} = 2\mathbf{a} + \mathbf{b} + \mathbf{a} + 2\mathbf{b}$
$= 3\mathbf{a} + 3\mathbf{b} = 3(\mathbf{a} + \mathbf{b}) = 3\overrightarrow{XY}$

b 3 : 1

c They lie on a straight line.

d Points are A(6, 2), B(1, 1) and C(2, 24). Using Pythagoras' theorem, $AB^2 = 26$, $BC^2 = 26$ and $AC^2 = 52$ so $AB^2 + BC^2 = AC^2$ hence ∠ABC must be a right angle.

12 In parallelogram ABCD, $\overrightarrow{AB} = \overrightarrow{DC} = \mathbf{a}$, $\overrightarrow{BC} = \overrightarrow{AD} = \mathbf{b}$. Let X be the midpoint of diagonal AC. Then $\overrightarrow{DX} = -\mathbf{b} + \frac{1}{2}(\mathbf{a} + \mathbf{b}) = \frac{1}{2}(\mathbf{a} - \mathbf{b}) = \frac{1}{2}\overrightarrow{DB}$ which is $\mathbf{a} - \mathbf{b}$, hence the midpoint of one diagonal is the same as the midpoint of the other diagonal, hence they bisect each other.

Review questions

1 **a** $2\mathbf{b} - \mathbf{a}$ **b** $-3\mathbf{b}$ **c** $\mathbf{a} + \mathbf{b}$

2 **a i** $2\mathbf{y} - 2\mathbf{b}$ **ii** $2\mathbf{b} - 2\mathbf{x}$

b $\overrightarrow{WZ} = \overrightarrow{WB} + \overrightarrow{BZ} = \frac{1}{2}(2\mathbf{b} - 2\mathbf{x}) + \frac{1}{2}(2\mathbf{y} - 2\mathbf{b})$
$= \mathbf{b} - \mathbf{x} + \mathbf{y} - \mathbf{b} = \mathbf{y} - \mathbf{x}$

c $\overrightarrow{XY} = \mathbf{y} - \mathbf{x}$, so parallel and equal in length to WZ, so Tim must be correct.

3 Let $\overrightarrow{OF} = \mathbf{x} = \overrightarrow{DE}$ and $\overrightarrow{OD} = \mathbf{y} = \overrightarrow{FE}$, then $\overrightarrow{DF} = \mathbf{x} - \mathbf{y}$ and $\overrightarrow{AB} = \frac{1}{2}\mathbf{x} - \frac{1}{2}\mathbf{y} = \frac{1}{2}(\mathbf{x} - \mathbf{y})$

4 **a i** $\mathbf{p} + \mathbf{r}$ **ii** $\mathbf{r} - \mathbf{p}$

b $\frac{1}{2}(\mathbf{r} - \mathbf{p})$

c $\overrightarrow{OX} = \mathbf{p} + \frac{1}{2}(\mathbf{r} - \mathbf{p}) = \frac{1}{2}(\mathbf{p} + \mathbf{r}) = \frac{1}{2}\overrightarrow{OQ}$

5 **a i** $\mathbf{y} - \mathbf{x}$ **ii** $\frac{1}{2}(\mathbf{y} - \mathbf{x})$ **iii** $\frac{1}{2}(\mathbf{x} + \mathbf{y})$

iv $\frac{1}{3}(\mathbf{x} + \mathbf{y})$ **v** $-\frac{1}{6}(\mathbf{x} + \mathbf{y})$ **vi** $\frac{1}{3}(\mathbf{y} - 2\mathbf{x})$

vii $\frac{1}{2}\mathbf{y} - \mathbf{x}$

b $\overrightarrow{BG} = \frac{1}{3}(\mathbf{y} - 2\mathbf{x})$ and $\overrightarrow{BE} = \frac{1}{2}\mathbf{y} - \mathbf{x}$
$= \frac{1}{2}(\mathbf{y} - 2\mathbf{x})$, both are multiples of $(\mathbf{y} - 2\mathbf{x})$ so are parallel, and with a common point, they must all be collinear.

c $\frac{2}{3}\overrightarrow{BE} = \frac{2}{3} \times \frac{1}{2}(\mathbf{y} - 2\mathbf{x}) = \frac{1}{3}(\mathbf{y} - 2\mathbf{x}) = \overrightarrow{BG}$

6 **a i** $\mathbf{x} + \mathbf{y}$ **ii** $\frac{1}{3}(\mathbf{x} + \mathbf{y})$ **iii** $\frac{2}{3}(\mathbf{x} + \mathbf{y})$

iv $\frac{1}{3}(\mathbf{x} - 2\mathbf{y})$ **v** $\frac{1}{3}(\mathbf{x} - 2\mathbf{y})$

b $\overrightarrow{SA} = \frac{1}{3}(\mathbf{x} - 2\mathbf{y}) = \overrightarrow{BQ}$, $\overrightarrow{SB} = \overrightarrow{SA} + \overrightarrow{AB}$, $\overrightarrow{AQ} = \overrightarrow{AB} + \overrightarrow{BQ} = \overrightarrow{AB} + \overrightarrow{SA}$, hence $\overrightarrow{SB} = \overrightarrow{AQ}$, hence SAQB has opposite sides equal and parallel, so a parallelogram.

7 **a i** $6\mathbf{a} - 2\mathbf{b}$ **ii** $3\mathbf{a} - \mathbf{b}$

b $\overrightarrow{BP} = 2(3\mathbf{a} - \mathbf{b})$ hence it is parallel to BQ with a common point Q, so the points B, Q and P are collinear.

8 In triangle ABC the midpoint of AB is M and the midpoint of AC is N

Let $\overrightarrow{AM} = \mathbf{x}$ and $\overrightarrow{AN} = \mathbf{y}$, then $\overrightarrow{MN} = \mathbf{y} - \mathbf{x}$, AB = $2\mathbf{x}$ and AC = $2\mathbf{y}$, so $\overrightarrow{BC} = 2\mathbf{y} - 2\mathbf{x} = 2(\mathbf{y} - \mathbf{x})$. $\overrightarrow{BC}$ is a multiple of $\overrightarrow{MN}$ and so parallel, $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{BC}$ and so half its length.

9 **a** $\mathbf{m} - \mathbf{r}$

b $\overrightarrow{RT} = 2\overrightarrow{RM} = 2(\mathbf{m} - \mathbf{r})$, so $\overrightarrow{NT} = -2\mathbf{m} + 2\mathbf{m} - 2\mathbf{r} = -2\mathbf{r}$, parallel to $\mathbf{r}$, hence NT is parallel to OR.